VKR Classes

VKR Sir
B.Tech., IIT DELHI
with you since 15 years

JEE Main

Time : 1 hr. Test Paper 08 Date 04/01/15 Batch - R Marks : 120

SINGLE CORRECT CHOICE TYPE [4, –1]

1. If the complex number z satisfies the condition $|z| \geq 3$, then the least value of $\left|z + \frac{1}{z}\right|$ is equal to :

(A) 5/3 (B) 8/3 (C) 11/3 (D) none of these

2. The integral, $\int_{\pi/4}^{5\pi/4} (|\cos t|\sin t + |\sin t|\cos t)\,dt$ has the value equal to

(A) 0 (B) 1/2 (C) $1/\sqrt{2}$ (D) 1

3. A curve is represented parametrically by the equations $x = t + e^{at}$ and $y = -t + e^{at}$ when $t \in R$ and $a > 0$. If the curve touches the axis of x at the point A, then the coordinates of the point A are

(A) (1, 0) (B) (1/e, 0) (C) (e, 0) (D) (2e, 0)

4. If $z = x + iy$ & $\omega = \frac{1 - iz}{z - i}$ then $|\omega| = 1$ implies that, in the complex plane :

(A) z lies on the imaginary axis (B) z lies on the real axis

(C) z lies on the unit circle (D) none

5. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3, 4 and 5 sq. units respectively. Then the area of the triangle BCD, is

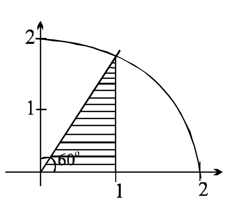
(A) $5\sqrt{2}$ (B) 5 (C) $\frac{5}{\sqrt{2}}$ (D) $\frac{5}{2}$

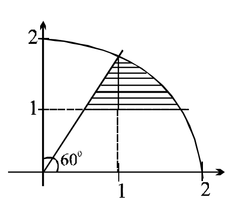
6. Let C_1 and C_2 are concentric circles of radius 1 and 8/3 respectively having centre at (3, 0) on the argand plane. If the complex number z satisfies the inequality, $\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$ then :

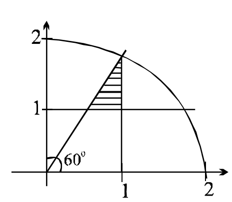
(A) z lies outside C_1 but inside C_2 (B) z lies inside of both C_1 and C_2

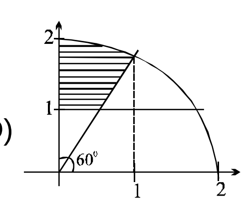
(C) z lies outside both of C_1 and C_2 (D) none of these

7. The region represented by inequalities $\text{Arg } Z \leq \frac{\pi}{3}$; $|Z| \leq 2$; $\text{Im}(z) \geq 1$ in the Argand diagram is given by

(A) 

(B) 

(C) 

(D) 

8. Number of roots of the function $f(x) = \frac{1}{(x+1)^3} - 3x + \sin x$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

9. A beam of light is sent along the line x – y = 1. Which after refracting from the x-axis enter the opposite side by turning through 30º towards the normal at the point of incidence on the x-axis. The equation of the refracted ray is

(A) $(2 + \sqrt{3})x - y = 2 + \sqrt{3}$ (B) $(2 + \sqrt{2})x - y = 2 + \sqrt{2}$

(C) $\sqrt{3}x - y = \sqrt{3}$ (D) None of these

10. If a, c, b are in G.P. then the line ax + by + c = 0 :

(A) has a fixed direction

(B) always passes through a fixed point

(C) forms a traingle with the axes whose area is constant

(D) always cuts intercepts on the axes such that their sum is zero

11. Let f be a real valued function with derivatives upto order two for all $x \in R$. If $\ell n\left(\frac{f(b) + f'(b)}{f(a) + f'(a)}\right) = b - a$ for real number a & b where a < b then there is a number $c \in (a, b)$ for which

(A) $f''(c) = e^{f(c)}$ (B) $f''(c) = f(c)$ (C) $f''(c) = cf(c)$ (D) None of these

12. Write the correct sequence of True & False for the following:

(i) At the point (0, 1), the tangent line to $y = \frac{1}{1+x^2}$ has the greatest slope.

(ii) $f(x) = \int_0^x t(t-1)(t-2)\,dt$ takes on its manimum value at x = 0, 2.

(iii) The maximum value of $x + \frac{1}{x}$ is less than its minimum value.

(iv) The maximum value of $x^{1/x}$ is $e^{1/e}$, (x > 0)

(A) FFTT (B) TTFT (C) FTTT (D) none

13. Let A and B be n × n matrices over the reals. If $A_0B = A + B - AB$ then $A_0B = B_0A = O$ if and only if :-

(A) A is non-singular (B) B is non-singular

(C) $(I_n - A)$ is non-singular (D) None of these

14. The complex number ω satisfying the equation $\omega^3 = 8i$ and lying in the second quadrant on the complex plane is

(A) $-\sqrt{3} + i$ (B) $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ (C) $-2\sqrt{3} + i$ (D) $-\sqrt{3} + 2i$

15. The number of points with integral coordinates lying in the interior of the quadilateral formed by lines 2x + y – 2 = 0, 4x + 5y = 20 and the coordinate axes is -

(A) 5 (B) 6 (C) 7 (D) none of these

16. The true set of real values of λ for which the point P with co-ordiante (λ, λ^2) does not lie inside the triangle formed by the lines, x – y = 0 ; x + y – 2 = 0 & x + 3 = 0 is -

(A) $(-\infty, -2]$ (B) $[0, \infty)$ (C) $[-2, 0]$ (D) $(-\infty, -2] \cup [0, \infty)$

17. Number of imaginary complex numbers satisfying the equation, $z^2 = \bar{z}\,2^{1-|z|}$ is

(A) 0 (B) 1 (C) 2 (D) 3

18. Two opposite sides of rhombus are x + y = 1 and x + y = 5. If one vertex is (2, – 1) and the angle at the vertex is 45º, a vertex opposite to the given vertex is.

(A) $(6 + 2\sqrt{2}, -1-2\sqrt{2})$ (B) $(6 - 2\sqrt{2}, 1 + 2\sqrt{2})$

(C) $(6 - 2\sqrt{2}, 1-2\sqrt{2})$ (D) none of these

19. If f(x) = sgn($\sin^2 x - \sin x - 1$) has exactly four points of discontinuity for $x \in (0, n\pi)$, $n \in N$ then

(A) the minimum value of n is 5 (B) the maximum value of n is 6

(C) there are exactly two possible values of n (D) none of these

20. Let $\int \frac{dx}{x^{2008}+x} = \frac{1}{p}\ell n\left(\frac{x^q}{1+x^r}\right) + C$ where p, q, r $\in$ N and need not be distinct, then the value (p + q + r) equals

(A) 6024 (B) 6022 (C) 6021 (D) 6020

21. Let $f(x) = e^{e^{e^x}}$, denotes $f'(0) = l$,

$g(x) = x \ln x + x$, denotes $g'(e) = m$,

$h(x) = \frac{d}{dx}\int_0^x t\,dt$, denotes $h'(1) = n$,

then the value of $\frac{lmn}{e^e}$, is

(A) 3 (B) 3e (C) $3e^e$ (D) $e \cdot e^e$

22. Consider two functions f (x) = sin x and g (x) = | f (x) |.

Statement-1 : The function h (x) = f (x) g(x) is not differentiable in [0, 2π]

Statement-2 : f (x) is differentiable and g (x) is not differentiable in [0, 2π]

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

23. In a quadrilateral ABCD, $\vec{AC}$ is the bisector of the $\left(\vec{AB} \wedge \vec{AD}\right)$ which is $\frac{2\pi}{3}$, $15\left|\vec{AC}\right| = 3\left|\vec{AB}\right| = 5\left|\vec{AD}\right|$ then $\cos\left(\vec{BA} \wedge \vec{CD}\right)$ is :

(A) $-\frac{\sqrt{14}}{7\sqrt{2}}$ (B) $-\frac{\sqrt{21}}{7\sqrt{3}}$ (C) $\frac{2}{\sqrt{7}}$ (D) $\frac{2\sqrt{7}}{14}$

24. If the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ is decomposed into vectors parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ then the vectors are :

(A) $-(\hat{i}+\hat{j}+\hat{k})$ & $7\hat{i}-2\hat{j}-5\hat{k}$ (B) $-2(\hat{i}+\hat{j}+\hat{k})$ & $8\hat{i}-\hat{j}-4\hat{k}$

(C) $+2(\hat{i}+\hat{j}+\hat{k})$ & $4\hat{i}-5\hat{j}-8\hat{k}$ (D) none

25. The number of points, where the function f(x) = max (|tan x|, cos |x|) is non-differentiable in the interval (–π, π), is

(A) 4 (B) 6 (C) 3 (D) 2

26. If A (– 4, 0, 3) ; B (14, 2, –5) then which one of the following points lie on the bisector of the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ ('O' is the origin of reference)

(A) (2, 1, –1) (B) (2, 11, 5) (C) (10, 2, –2) (D) (1, 1, 2)

27. Suppose x_1 & x_2 are the point of maximum and the point of minimum respectively of the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ respectively, then for the equality $x_1^2 = x_2$ to be true the value of 'a' must be :

(A) 0 (B) 2 (C) 1 (D) none

28. The value of the $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{i}{n^2} \sin \frac{\pi i^2}{n^2}$ is

(A) 1 (B) $\frac{1}{\pi}$ (C) $\frac{2}{\pi}$ (D) $\frac{1}{2\pi}$

29. Four coplanar forces are applied at a point O . Each of them is equal to k , & the angle between two consecutive forces equals 45º . Then the resultant has the magnitude equal to :

(A) $k\sqrt{2+2\sqrt{2}}$ (B) $k\sqrt{3+2\sqrt{2}}$ (C) $k\sqrt{4+2\sqrt{2}}$ (D) none

30. If $f(x) = x^4 + ax^3 + bx^2 + cx + d$ be polynomial with real coefficient and real roots. If $|f(i)| = 1$, where $i = \sqrt{-1}$, then a + b + c + d is equal to

(A) – 1 (B) 1 (C) 0 (D) can not be determined

Answer Sheet

Student Name: ______________________________ **Batch : R** **Date : 04/01/15**

1. Ⓐ Ⓑ Ⓒ Ⓓ	2. Ⓐ Ⓑ Ⓒ Ⓓ	3. Ⓐ Ⓑ Ⓒ Ⓓ	4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ	6. Ⓐ Ⓑ Ⓒ Ⓓ	7. Ⓐ Ⓑ Ⓒ Ⓓ	8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ	10. Ⓐ Ⓑ Ⓒ Ⓓ	11. Ⓐ Ⓑ Ⓒ Ⓓ	12. Ⓐ Ⓑ Ⓒ Ⓓ
13. Ⓐ Ⓑ Ⓒ Ⓓ	14. Ⓐ Ⓑ Ⓒ Ⓓ	15. Ⓐ Ⓑ Ⓒ Ⓓ	16. Ⓐ Ⓑ Ⓒ Ⓓ
17. Ⓐ Ⓑ Ⓒ Ⓓ	18. Ⓐ Ⓑ Ⓒ Ⓓ	19. Ⓐ Ⓑ Ⓒ Ⓓ	20. Ⓐ Ⓑ Ⓒ Ⓓ
21. Ⓐ Ⓑ Ⓒ Ⓓ	22. Ⓐ Ⓑ Ⓒ Ⓓ	23. Ⓐ Ⓑ Ⓒ Ⓓ	24. Ⓐ Ⓑ Ⓒ Ⓓ
25. Ⓐ Ⓑ Ⓒ Ⓓ	26. Ⓐ Ⓑ Ⓒ Ⓓ	27. Ⓐ Ⓑ Ⓒ Ⓓ	28. Ⓐ Ⓑ Ⓒ Ⓓ
29. Ⓐ Ⓑ Ⓒ Ⓓ	30. Ⓐ Ⓑ Ⓒ Ⓓ		

ANSWER WITH SOLUTION

JEE MAINS						ANSWER KEY						
Q.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
A.	B	A	D	B	A	A	B	C	A	C	B	AorC
Q.	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
A.	C	A	B	D	C	A	C	C	B	D	C	A
Q.	**25**	**26**	**27**	**28**	**29**	**30**						
A.	A	D	B	B	C	C						

SOLUTION

2. $I = \int_{\pi/4}^{\pi/2} 2\sin t\cos t\,dt + \int_{\pi/2}^{\pi} \underbrace{(-\sin t\cos t)+(\sin t\cos t)}_{\text{zero}}dt + \int_{\pi}^{5\pi/4} -2\sin t\cos t\,dt$

$= \int_{\pi/4}^{\pi/2} \sin 2t\,dt - \int_{\pi}^{5\pi/4} \sin 2t\,dt$

these two integrals cancels $\Rightarrow$Zero]

3. $x = t + e^{at}$; $y = -t + e^{at}$

$\frac{dx}{dt} = 1 + ae^{at}$; $\frac{dy}{dt} = -1 + ae^{at}$; $\frac{dy}{dx} = \frac{-1+ae^{at}}{1+ae^{at}}$

at the point A, $y = 0$ and $\frac{dy}{dx} = 0$ for some $t = t_1$

$\therefore$ $ae^{at_1} = 1$ ....(1) ;

also $0 = -t_1 + e^{at_1}$; $\therefore$ $e^{at_1} = t_1$ .....(2), putting this value in (1)

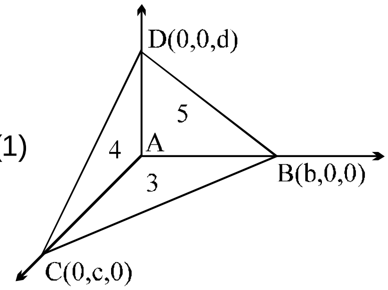
we get, $at_1 = 1$ $\Rightarrow$ $t_1 = \frac{1}{a}$; now from (1) $ae = 1$ $\Rightarrow a = \frac{1}{e}$

hence $x_A = t_1 + e^{at_1} = e + e = 2e \Rightarrow$ $A \equiv (2e, 0)$ Ans.]

5. Area of Δ BCD = $\frac{1}{2}\left|\overrightarrow{BC}\times\overrightarrow{BD}\right| = \frac{1}{2}\left|(b\hat{i}-c\hat{j})\times(b\hat{i}-d\hat{k})\right|$

$= \frac{1}{2}\left|bd\,\hat{j}+bc\,\hat{k}+dc\,\hat{i}\right|$

$= \frac{1}{2}\sqrt{b^2c^2+c^2d^2+d^2b^2}$ ....(1)



now $6 = bc$; $8 = cd$; $10 = bd$

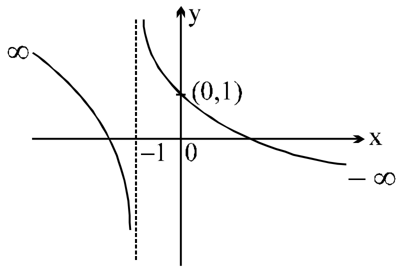
$b^2c^2 + c^2d^2 + d^2b^2 = 200$

substituting the value in (1)

$A = \frac{1}{2}\sqrt{200} = 5\sqrt{2}$ Ans.]

8. $f'(x) = -\frac{3}{(x+1)^4} - 3 + \cos x < 0$

hence f (x) is always decreasing, Also as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to +\infty$
hence one positive and one negative root
Graph is as shown



11. consider $g(x) = (f(x) + f'(x))e^{-x}$
From the given information, g(a) = g(b).
By Rolle's Theorem, there exists $c \in (a, b)$
such that $g'(c) = 0$.
Here $g'(x) = (f''(x) - f(x))e^{-x}$
$g'(c) = 0 \Rightarrow f''(c) = f(c)$

12. (i) (F)

$y = \frac{1}{1+x^2} \Rightarrow \frac{dy}{dx} = -\frac{2x}{(1+x^2)^2} = 0$ at (0, 1) $\Rightarrow \tan\Psi = 0 \Rightarrow \Psi = 0$

$\therefore$ slope is not greatest.
(iii) (T)

$y = x + \frac{1}{x}$ ....(1) $\Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$ ....(2)

$\frac{d^2y}{dx^2} = \frac{2}{x^3}$ ....(3)

$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{1}{x^2} = 0 \Rightarrow x = 1, x = -1 \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=1} > 0, \left(\frac{d^2y}{dx^2}\right)_{x=-1} < 0$

$\therefore$ y is max. if x = –1 . y is min. at x = 1
$\Rightarrow$ (max). (y) = 1 – 1 = 0, min.(y) = 1 + 1 = 2 $\Rightarrow$ max. value < min. value]

13. $A_0B = O$ if and only if $(I_n - A)(I_n - B) = I_n$. This implies that $I_n - A$ is non-singular. Conversly, if $I_n - A$ is non-singular and let C be its inverse and let $B = I_n - C$ then $C = I_n - B$. So $(I_n - A)(I_n - B) = I_n$

20. $I = \int \frac{dx}{x(x^{2007}+1)} = \int \frac{x^{2007}+1-x^{2007}}{x(x^{2007}+1)}dx = \int \left(\frac{1}{x} - \frac{x^{2006}}{1+x^{2007}}\right)dx$

$= \ell n\, x - \frac{1}{2007}\ln(1 + x^{2007}) = \frac{\ell n x^{2007} - \ell n(1+x^{2007})}{2007} = \frac{1}{2007}\ell n\left(\frac{x^{2007}}{1+x^{2007}}\right) + C$

p + q + r = 6021 Ans.

21. $l = e \cdot e^e$; m = 3; n = 1 $\Rightarrow \frac{lmn}{e^e} = 3e$ Ans.]

22. f (x) = sin x is differentiable in [0, 2π]
g (x) = | sin x | is not differentiable at x = π.
Let h (x) = f (x) g (x) = | sin x | sin x

$h'(\pi) = \lim_{h\to 0} \frac{|\sin(\pi - h)|\sin(\pi-h) - 0}{h} = \lim_{h\to 0} \frac{|\sin h|\sin h}{h} = 0$

$\Rightarrow$ h (x) is differentiable at x = π

but g (x) is not differentiable at x = π]

26. $\overrightarrow{OA} = -4\hat{i} + 3\hat{k}$; $\overrightarrow{OB} = 14\hat{i} + 2\hat{j} - 5\hat{k}$

$\hat{a} = \frac{-4\hat{i} + 3\hat{k}}{5}$; $\hat{b} = \frac{14\hat{i} + 2\hat{j} - 5\hat{k}}{15}$

$\vec{r} = \frac{\lambda}{15}\left[-12\hat{i} + 9\hat{j} + 14\hat{i} + 2\hat{j} - 5\hat{k}\right]$

$\vec{r} = \frac{\lambda}{15}\left[2\hat{i} + 2\hat{j} + 4\hat{k}\right]$

$\vec{r} = \frac{2\lambda}{15}\left[\hat{i} + \hat{j} + 2\hat{k}\right]$]

28. $T_r = \frac{r}{n^2}\cdot\sin\frac{\pi r^2}{n^2} = \frac{1}{n}\cdot\frac{r}{n}\sin\pi\left(\frac{r}{n}\right)^2$

Sum = $\sum T_r = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n}\sin\pi\left(\frac{r}{n}\right)^2 = \int_0^1 x\sin\pi x^2\,dx$

put $\pi x^2 = t$ $\Rightarrow$ $2\pi x\,dx = dt$

S = $\frac{1}{2\pi}\int_0^{\pi}\sin t\,dt = \frac{1}{\pi}$ Ans.

30. Let $f(x) = (x - x_1)(x - x_2)(x - x_3)(x - x_4)$

$|f(i)| = \sqrt{1+x_1^2}\ \sqrt{1+x_2^2}\ \sqrt{1+x_3^2}\ \sqrt{1+x_4^2} = 1$

$\Rightarrow$ $x_1 = x_2 = x_3 = x_4 = 0$ $\Rightarrow$ all four roots are zero $\Rightarrow$ $f(x) = x^4$

$\therefore$ a = b = c = d = 0 $\Rightarrow$ a + b + c + d = 0 $\Rightarrow$ (C)]
