VKR Classes

VKR Sir
B.Tech., IIT DELHI
with you since 15 years

JEE Advanced

Time : 2 hr. Test Paper 12 Date 25/01/15 Batch - P Marks : 120

SINGLE CORRECT CHOICE TYPE [3, –1]

1. The domain of definition of the function $f(x) = \log\left(\sqrt{10\cdot3^{x-2} - 9^{x-1} - 1}\right) + \sqrt{\cos^{-1}(1-x)}$ is

(A) [0, 1] (B) [1, 2] (C) (0, 2) (D) (0, 1)

2. If f(x) is even, periodic function defined for all $x \in R$ and has period 1then

(A) $f\left(x+\frac{1}{2}\right) = f(x)$ (B) $f\left(\frac{1}{3}+x\right) = f\left(\frac{2}{3}-x\right)$ (C) f(x+ 1) = f(2x + 1) (D) f(0) can not be zero

3. The line $(k + 1)^2x + ky - 2k^2 - 2 = 0$ passes through a point regardless of the value k. Which of the following is the line with slope 2 passing through the point?

(A) y = 2x – 8 (B) y = 2x – 5 (C) y = 2x – 4 (D) y = 2x + 8

4. If a, b, c are non-zero real numbers, then the minimum value of the expression

$\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right)$ equals

(A) 125 (B) 315 (C) 343 (D) 729

5. The number k is such that tan {(arc tan (2) + arc tan(20k)} = k. The sum of all possible values of k is

(A) $-\frac{19}{40}$ (B) $-\frac{21}{40}$ (C) 0 (D) $\frac{1}{5}$

6. A class has three teachers, Mr. P, Ms. Q and Mrs. R and six students A,B,C,D,E,F. Number of ways in which they can be seated in a line of 9 chairs, if between any two teachers there are exactly two students, is k(6!) then the value of k is

(A) 18 (B) 12 (C) 24 (D) 6

7. The number of ways in which a mixed double tennis game can be arranged from amongst 9 married couple if no husband & wife plays in the same game is :

(A) 756 (B) 1512 (C) 3024 (D) 4536

8. If $\sin^4x + \cos^4y + 2 = 4 \sin x \cos y$, and $0 \le x, y \le \frac{\pi}{2}$ then sin x + cos y is equal to

(A) –2 (B) 0 (C) 2 (D) none of these

COMPREHENSION [3, -1]

Comprehension # 1

Consider the family of circles $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ passing through two fixed points A and B. Also S = 0 is a circle of this family, the tangent to which at A and B intersect on the line x + 2y+ 5 = 0 .

9. The distance between the points A and B, is

(A) 4 (B) $4\sqrt{2}$ (C) 6 (D) 8

10. If the circle $x^2 + y^2 - 10x + 2y + c = 0$ is orthogonal to S = 0, then the value of c equals

(A) 8 (B) 9 (C) 10 (D) 12

Comprehension # 2

Let P(x) be a polynomial such that P(1) = 1 and $\frac{P(2x)}{P(x+1)} = 8 - \frac{56}{x+7}$ for all real x for which both sides are defined.

11. The value of P(–1) is

(A) $\frac{2}{7}$ (B) $-\frac{4}{21}$ (C) $-\frac{5}{21}$ (D) $-\frac{11}{21}$

12. The number of real roots of P(x) = 0 is

(A) 5 (B) 3 (C) 2 (D) None of these

Comprehension # 3

In a sequence of (4n + 1) terms the first(2n + 1) terms are in AP whose common difference is 2, and the last (2n + 1) terms are in GP whose common ratio 0.5. If the middle terms of the AP and GP are equal then

13. The middle term of the sequence is

(A) $\frac{n.2^{n+1}}{2^n - 1}$ (B) $\frac{n.2^{n+1}}{2^{2n} - 1}$ (C) $n.2^n$ (D) None of these

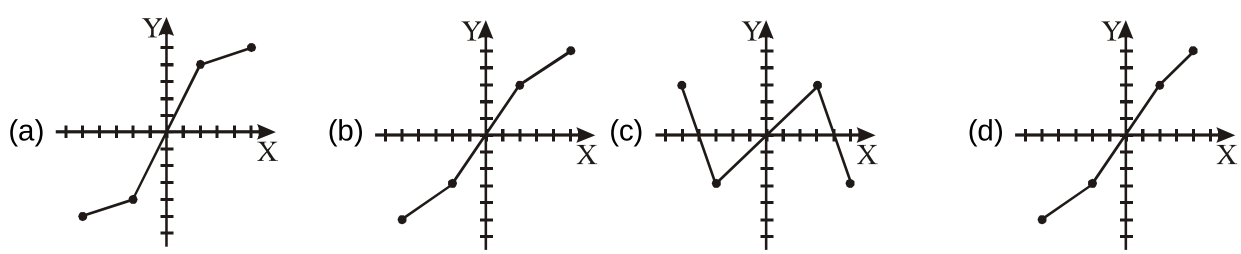
14. The middle term of the GP is

(A) $\frac{2^n}{2^2 - 1}$ (B) $\frac{n.2^n}{2^2 - 1}$ (C) $\frac{n}{2^n - 1}$ (D) $\frac{2n}{2^n - 1}$

Comprehension # 4

Consider the function f : $y = f(x) = \begin{cases} \frac{x}{3} - \frac{10}{3} & \text{if } x \in [-5, -2) \\ 2x & \text{if } x \in [-2, 2] \\ \frac{x}{3} + \frac{10}{3} & \text{if } x \in (2, 5] \end{cases}$

15. Which one most resembles the graph of f ?



(A) a (B) b (C) c (D) d

16. Which one could not possibly be a possible value for $\underbrace{(f \circ \ldots \circ f)}_{\text{n compositions}}(a)$, where n is a positive integer and a $\in [-5, 5]$?

(A) 0 (B) –5 (C) 5 (D) 6

MULTIPLE CORRECT CHOICE TYPE [3, 0]

17. If the equation $\sin^{-1}(x^2 + x + 1) + \cos^{-1}(ax + 1) = \frac{\pi}{2}$ has exactly two solutions then a cannot have the value

(A) –1 (B) 0 (C) 1 (D) 2

18. The equation $\left(\frac{x}{x+1}\right)^2 + \left(\frac{x}{x-1}\right)^2 = a(a-1)$ has

(A) four real roots if $a > 3$
(B) has two realroots if $1 < a < 2$
(C) has no real roots if $a < -1$
(D) has four real roots for all $a < -1$

19. Let a, b, c be the roots of $x^3 - 9x^2 + 11 - 1 = 0$ and let $s = \sqrt{a} + \sqrt{b} + \sqrt{c}$ and $t = \sqrt{ab} + \sqrt{bc} + \sqrt{ca}$, then

(A) $t^2 = 11 + 2s$
(B) $s^4 = 125 + 36t + 8s$
(C) $s^4 - 18s^2 - 8s = -37$
(D) $s^4 - 18s^2 + 8s = 37$

20. Let $f : \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \to [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$. then $f^{-1}(x)$ is given by

(A) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$ (C) $\frac{2\pi}{3} - \cos^{-1}\left(\frac{x-2}{2}\right)$ (D) None of these

21. If $f(x) = \sin(\{x\} + \sin ax)$ is periodic with period '1' then 'a' may be equal to (where $\{x\}$ denotes fractional part of x):

(A) 0 (B) 2π (C) 4π (D) π

22. Let $f(x) = x^3 - 2x$, $g(x) = x^3 - x|x|$, $h(x) = 3x^2 - |x| + 6$, $k(x) = x^4 - 2x + |x + 1|$. Which of the following functions is either odd or even?

(A) kog (B) koh (C) fog (D) fohog

23. If $f: R \to R$ is defined as $f(x) = e^{|x|} - e^{-x}$, then the correct statement(s) is / are

(A) f is one - one onto function
(B) f is many one into function
(C) range of f is $[0, \infty)$
(D) range of f is $(-\infty, \infty)$

24. In a triangle ABC, altitude from its vertex meet the opposite sides in D, E and F. Then the perimeter of the triangle DEF, is

(A) $\frac{abc}{4R^2}$ (B) $\frac{2\Delta}{R}$ (C) $\frac{R(a+b+c)}{r}$ (D) $\frac{2rs}{R}$

25. Consider the circle $x^2 + y^2 - 8x - 18y + 93 = 0$ with centre 'C' and point P(2, 5) outside it. From the point P, a pair of tangents PQ and PR are drawn to the circle with S as the midpoint of QR. The line joining P to C intersects the given circle at A and B. Which of the following hold(s) good?

(A) CP is the arithmetic mean of AP and BP.

(B) PR is the geometric mean of PS and PC.

(C) PS is the harmonic mean of PAand PB.

(D) The angle between the two tangents from P is $\tan^{-1}\left(\frac{3}{4}\right)$.

26. if $a = \underbrace{111.......1}_{55\text{ times}}$, $b = 1 + 10 + 10^2 + 10^3 + 10^4$ and $c = 1 + 10^5 + 10^{10} + ...+10^{50}$ then

(A) b, $\frac{a}{2}$, c are in A.P.
(B) b, $\sqrt{a}$, c are in G.P.
(C) a is a prime number
(D) a is a composite number

where Δ is the area of the triangle ABC and all other symbols have their usual meaning.

INTEGER ANSWER TYPE [3, 0]

27. The sum of the squares of all the solution(s) of the equation, $2\sin^{-1}(x+2) = \cos^{-1}(x+3)$ is

28. Suppose the domain of the function $y = f(x)$ is $-1 \le x \le 4$ and the range is $1 \le y \le 10$. Let $g(x) = 4 - 3f(x-2)$. If the domain of g(x) is $a \le x \le b$ and the range of g(x) is $c \le y \le d$ then the value of $(a + b + d)$ is

29. Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

30. If $f(\theta) = \dfrac{1-\sin 2\theta+\cos 2\theta}{2\cos 2\theta}$ then the reciprocal of the value of $f(11°) \cdot f(34°)$ equals

31. The least integral value of k for which $(k-2)x^2 + 8x + k + 4 > \sin^{-1}(\sin 12) + \cos^{-1}(\cos 12)$ for all $x \in R$, is

32. The number of solutions of the equation $|[x] - 2x| = 4$, where [x] is the greatest integer $\le x$, is equal to2

33. If the equation $\sec\theta + \operatorname{cosec}\theta = c$ has two real roots between 0 and 2π, then the least integer which c^2 cannot exceed is equal to

34. If the sum to first *n* terms of a series, the r^{th} term of which is given by $(2r + 1)2^r$ can be expressed as $R(n \cdot 2^n) + S \cdot 2^n + T$, then find the value of (R + S + T).

35. Let α, β and γ be three distinct real roots of the equation $x(3x + 2)^2 + 2 = (a + 12 + 9x)x^2 - bx + c$ where a, b, $c \in R$. If every solution of the inequality $(x - a)^2 (4x + b) (x - c) < 0$ is also solution of the inequality $3x^2 + px + p^2 + 6p < 0$ then find number of integral values of 'p'.

36. The number of solutions of the equation $\log^2(4-x) + \log(4-x) \cdot \log\left(x+\frac{1}{2}\right) - 2\log^2\left(x+\frac{1}{2}\right) = 0$ is

MATRIX MATCH TYPE [3, –1]

37. Match the following

	Column - I	**Column - II**
(A)	In a triangle ABC, a = 7, b = 8, c = 9, BD is median, BE is aititude, then ED is equal to	(P) 2
(B)	If $\frac{3}{5}$ th of all the 'three element subsets' of A = $\{a_1, a_2, ... a_n\}$ contain the element a_n, then n is equal to	(Q) 3
(C)	In a triangle ABC, $\sin A \cos B = \frac{1}{4}$ and $3\tan A = \tan B$, then $\cot^2 A$ si equal to	(R) 4
(D)	Out fo 10 pens 4 are identical of black colour, some are identical of white colout and rest are diferent. If they can be arranged in 6300 ways, then the number of white colout pens is equal to	(S) 5

38.

	Column I	**Column II**
(A)	The minimum value of $\|x - p\| + \|x - 15\| + \|x - p - 15\|$ for x is the range $p \le x \le 15$, where $0 < p < 15$.	(P) 25
(B)	The number of 4 digit number with first digit 1 having exactly two identical digits is 36 k where k is	(Q) 15
(C)	The function f: $Z \to Z$ satisfies $f(x) = n - 3$ for $n > 999$ and $f(n) = f(f(n + 5))$ for $n < 100$. If $f(84) = k$ then the sum of the digit of k is	(R) 12
(D)	The number of rational terms in $(\sqrt[8]{5} + \sqrt[6]{2})^{100}$	(S) 4

39.

	Column-I		Column-II
(A)	If {(1, 1), (4, 2) and R (x, 0) be three point such that PR + RQ is minimum, then x is equal to	(P)	6
(B)	The area bounded by the curves max {|x|, |y|} = 1 is equal to	(Q)	2
(C)	The number of circles that touch all the three lines $2x - y = 5$, $x + y = 3$ and $4x - 2y = 7$ is equal to	(R)	3
(D)	If a line segment between the lines $3x + 2y - 15 = 0$ and $x + 2y - 4 = 0$ is bisected by the point (3, 1), then negative reciprocal of the slope of line containing the segment is	(S)	4

40.

	Column-I		Column-II
(A)	If three unequal numbers a, b, c are A.P. and b-c, c-b a-b are in G.P., then $\frac{a^3 + b^3 + c^3}{3abc}$ is equal to	(P)	$\frac{1}{3}$
(B)	Let x be the arithmetic mean and y,z be two goemetric means between any two positive numbers, then $\frac{y^3 + z^3}{xyz}$ is equal to	(Q)	1
(C)	If a, b, c be three positive number which form three successive terms of a G.P. and c > 4b – 3a, then the common ratio of the G.P. can be equal to	(R)	2
(D)	$\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{2r^2}\right)\right\}$ is equal to	(S)	3

Answer Sheet

Student Name: ______________________________ **Batch : P** **Date : 25/01/15**

1. Ⓐ Ⓑ Ⓒ Ⓓ	2. Ⓐ Ⓑ Ⓒ Ⓓ	3. Ⓐ Ⓑ Ⓒ Ⓓ	4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ	6. Ⓐ Ⓑ Ⓒ Ⓓ	7. Ⓐ Ⓑ Ⓒ Ⓓ	8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ	10. Ⓐ Ⓑ Ⓒ Ⓓ	11. Ⓐ Ⓑ Ⓒ Ⓓ	12. Ⓐ Ⓑ Ⓒ Ⓓ
13. Ⓐ Ⓑ Ⓒ Ⓓ	14. Ⓐ Ⓑ Ⓒ Ⓓ	15. Ⓐ Ⓑ Ⓒ Ⓓ	16. Ⓐ Ⓑ Ⓒ Ⓓ
17. Ⓐ Ⓑ Ⓒ Ⓓ	18. Ⓐ Ⓑ Ⓒ Ⓓ	19. Ⓐ Ⓑ Ⓒ Ⓓ	20. Ⓐ Ⓑ Ⓒ Ⓓ
21. Ⓐ Ⓑ Ⓒ Ⓓ	22. Ⓐ Ⓑ Ⓒ Ⓓ	23. Ⓐ Ⓑ Ⓒ Ⓓ	24. Ⓐ Ⓑ Ⓒ Ⓓ
25. Ⓐ Ⓑ Ⓒ Ⓓ	26. Ⓐ Ⓑ Ⓒ Ⓓ		

27. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	28. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
29. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	30. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
31. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	32. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
33. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	34. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
35. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	36. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

37. A	B	C	D	38. A	B	C	D	39. A	B	C	D	40. A	B	C	D
Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ
Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ
Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ
Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ

ANSWER WITH SOLUTION

ANSWER KEY

Q.	1	2	3	4	5	6	7	8	9	10	11	12
A.	C	B	A	B	A	A	B	C	C	D	C	B
Q.	13	14	15	16	17	18	19	20	21	22	23	24
A.	A	D	A	D	ACD	ABD	ABC	BC	ABC	BCD	BC	BD
Q.	25	26	27	28	29	30	31	32	33	34	35	36
A.	ABC	BD	4	8	4	2	5	4	8	4	5	3
Q.	37.						38.					
A.	(A) – (P), B – (S), (C) – (Q), (D) – (R)						(A) – (Q), B – (R), (C) – (P), (D) – (S)					
Q.	39.						40.					
A.	(A) – (Q), B – (S), (C) – (Q), (D) – (P)						(A) – (R), B – (R), (C) – (P,R), (D) – (Q)					

SOLUTION

3. $(k + 1)^2x + ky = 2k^2 + 2$

$(k^2 + 2k + 1)x + ky = 2k^2 + 2$

$k^2(x - 2) + k(2x + y) + (x - 2) = 0$

$(k^2 + 1)(x - 2) + k(2x + y) = 0$

$(2x + y) + (k + 1/k)(x - 2) = 0$

Hence line passes through fixed point, $x = 2$ and $y = -4$]

4. We have $\left(\frac{a^4+3a^2+1}{a^2}\right)\left(\frac{b^4+5b^2+1}{b^2}\right)\left(\frac{c^4+7c^2+1}{c^2}\right)$

$= \left(a^2+\frac{1}{a^2}+3\right)\left(b^2+\frac{1}{b^2}+5\right)\left(c^2+\frac{1}{c^2}+7\right) = \left(\left(a-\frac{1}{a}\right)^2+5\right)\left(\left(b-\frac{1}{b}\right)^2+7\right)\left(\left(c-\frac{1}{c}\right)^2+9\right)$

Clearly minimum value occurs when a = b = 1 = c and minimum value = 5 × 7 × 9 = 315]

6. (i) T S S T S S T S S (ii) S T S S T S S T S (iii) S S T S S T S S T

Hence $3 \cdot (3!)6! = (18)6! \Rightarrow$ k = 18 Ans.]

7. ${}^9C_2 \,.\, {}^7C_2 \,.\, 2! = 1512$]

24. sides of pedal Δ are a cosA, b cosB, c cosC

P = a cosA + b cosB + c cosC

$= R\left(\sum \sin 2A\right) = 4R \sin A \cdot \sin B \cdot \sin C = \frac{4Rabc}{8R^3} = \frac{abc}{2R^2} = \frac{abc \cdot 4\Delta}{2Rabc} = \frac{2\Delta}{R}$

$\Rightarrow$ (B)

$= \frac{2rs}{R} \Rightarrow$ (D)]

27. 4

Let $\sin^{-1}(x + 2) = \alpha \Rightarrow x + 2 = \sin\alpha$

$\therefore$ $2\alpha = \cos^{-1}(x + 3)$

$\cos 2\alpha = x + 3 = (x + 2) + 1 = 1 + \sin\alpha$

$1 - 2\sin^2\alpha = 1 + \sin\alpha$

$\sin\alpha\,(1 + 2\sin\alpha) = 0$

$\Rightarrow$ $\sin\alpha = 0$ or $\sin\alpha = -1/2$

$\therefore$ $x = -2$ or $x = -2.5$ (rejected)

$\therefore$ $x^2 = 6.25$]

28. 8

for domain of g(x) $-1 \le x - 2 \le 4$ $\Rightarrow$ $1 \le x \le 6$

hence a = 1 and b = 6

for range, $1 \le y \le 10$

$\Rightarrow$ $1 \le f(x) \le 10$

$\Rightarrow$ $1 \le f(x-2) \le 10$

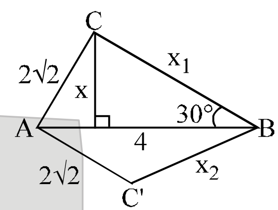
$\Rightarrow$ $3 \le 3f(x-2) \le 30$

$\therefore$ $-26 \le 4 - 3f(x-2) \le 1$

hence c = –26 and d = 1

29. [Ans. 4]

Apply cosine rule in ΔABC $(x \to x_1 \text{ or } x_2)$



$$\cos 30° = \frac{16 + x^2 - 8}{2 \times 4 \times x} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $x^2 + 8 - 4\sqrt{3}x = 0$

$$x = \frac{4\sqrt{3} \pm \sqrt{48 - 32}}{2} = \frac{4\sqrt{3} \pm 4}{2} = 2\sqrt{3} \pm 2$$

$\Rightarrow$ $x = 2\sqrt{3} + 2$ or $2\sqrt{3} - 2$

$$\left|\Delta ABC - \Delta ABC'\right| = \left|\frac{1}{2} \times 4\sin 30° \times BC - \frac{1}{2} \times 4\sin 30° \times BC'\right|$$

= BC – BC' = 4 **Ans.**]

30. Ans 2

$$f(\theta) = \frac{1 - \sin 2\theta + \cos 2\theta}{2\cos 2\theta} = \frac{(\cos\theta - \sin\theta)^2 + (\cos^2\theta - \sin^2\theta)}{2(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} = \frac{(\cos\theta - \sin\theta) + (\cos\theta + \sin\theta)}{2(\cos\theta + \sin\theta)}$$

$$= \frac{2\cos\theta}{2(\cos\theta + \sin\theta)} = \frac{1}{1 + \tan\theta}$$

$$f(11°) \cdot f(34°) = \frac{1}{(1 + \tan 11°)} \cdot \frac{1}{(1 + \tan 34°)} = \frac{1}{(1 + \tan 11°)} \cdot \frac{1}{(1 + \tan(45° - 11°))}$$

$$= \frac{1}{(1 + \tan 11°)} \cdot \frac{1}{1 + \frac{1 - \tan 11°}{1 + \tan 11°}} = \frac{1}{(1 + \tan 11°)} \cdot \frac{1 + \tan 11°}{2} = \frac{1}{2}$$ **Ans.**]

31. 5

$\sin^{-1}(\sin 12) = \sin^{-1}\sin(12 - 4\pi) = 12 - 4\pi$

$\cos^{-1}(\cos 12) = \cos^{-1}\cos(4\pi - 12) = 4\pi - 12$

$\therefore$ $(k-2)x^2 + 8x + k + 4 > 0$

if k = 2 then 8x + 4 > 0 (not possible)

and $64 - 4(k-2)(k+4) < 0$

$16 < k^2 + 2k - 8$

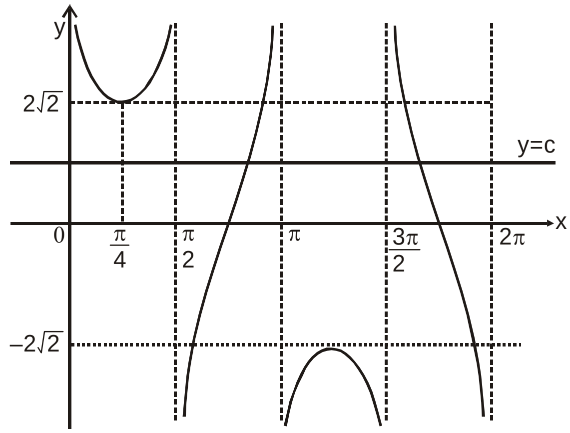
$k^2 + 2k - 24 > 0$

$(k+6)(k-4) > 0$

32. 4

33. **8**

Using graphical addition, the graph of $y = \sec\theta + \mathrm{cosec}\theta$ is shown



For two real roots $-2\sqrt{2} < c < 2\sqrt{2}$

$\Rightarrow \quad c^2 < 8$

34. [Ans. 4]

$T_r = (2r + 1)\, 2^r$

$S_n = 3\cdot2^1 + 5\cdot2^2 + 7\cdot2^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2n + 1)2^n$

Sub $\;2S_n = \qquad 3\cdot2^2 + 5\cdot2^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2n - 1)2^n + (2n + 1)2^{n+1}$

$-S_n = \underbrace{3\cdot2^1} + 2^3 + 2^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2^{n+1} - (2n + 1)2^{n+1}$

$\therefore \quad -S_n = \underbrace{2 + 2^2} + 2^3 + 2^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2^{n+1} - (2n + 1)2^{n+1}$

$\therefore \quad -S_n = \dfrac{2[2^{n+1} - 1]}{2 - 1} - (2n + 1)2^{n+1}$

$S_n = (2n + 1)2^{n+1} - (2^{n+2} - 2)$

$n\cdot2^{n+2} + \underbrace{2^{n+1} - 2^{n+2}} + 2$

$n\cdot2^{n+2} - \underbrace{2^{n+1}} + 2$

$4n \cdot 2^n - 2\cdot 2^n + 2$

$R = 4 \;;\; S = -2 \;;\; T = 2$

$\therefore \quad R + S + T = 4$ **Ans.**]

35. [Ans 5]

$x(3x + 2)^2 + 2 = (a + 12 + 9x)x^2 - bx + c$

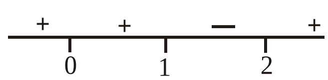
$\Rightarrow \quad 4x + 2 = ax^2 - bx + c$

which is quadratic in x but it satisfies by α, β and γ so it is an identity.

$\therefore$ a = 0, b = – 4, c = 2

$x^2 (4x - 4)(x - 2) < 0$

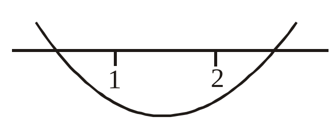
$\Rightarrow x^2 (x - 1)(x - 2) < 0$



$x \in (1, 2)$

$3x^2 + px + p^2 + 6p < 0$ satisfy by every $x \in (1, 2)$

$f(1) \le 0$ and $f(2) \le 0$



$p^2 + 7p + 3 \le 0$ and $p^2 + 8p + 12 \le 0$

$p \in \left[\dfrac{-7 - \sqrt{37}}{2}, \dfrac{-7 + \sqrt{37}}{2}\right] \qquad (p + 2)(p + 6) \le 0$

$\Rightarrow$ p $\in$ [–6, –2]

p $\in$ [–6.5... , –0...]   – 6, – 5, – 4, – 3, – 2

$\therefore$ Number of integers are 5 **Ans.**]

36. **3**

$\log^2 (4 - x) + \log (4 - x) . \log\left(x + \frac{1}{2}\right) - 2 \log^2\left(x + \frac{1}{2}\right) = 0$

let $\log(4 - x) = A$ & $\log\left(x + \frac{1}{2}\right) = B$

$A^2 + AB - 2B^2 = 0$

$A^2 + 2AB - AB - 2B^2 = 0$

$A(A + 2B) - B(A + 2B) = 0$

A = Bor A = – 2B

$\therefore$ $\log(4 - x) = \log\left(x + \frac{1}{2}\right)$

$\therefore$ $4 - x = x + \frac{1}{2} \Rightarrow 2x = \frac{7}{2} \Rightarrow x = \frac{7}{4}$

$\log(4 - x) = -2 \log\left(x + \frac{1}{2}\right)$

$4 - x = \frac{1}{(x + (1/2))^2} \Rightarrow (4 - x)(x^2 + \frac{1}{4} + x) = 1$

$4x^2 - x^3 + 1 - \frac{x}{4} + 4x - x^2 - 1 = 0$

$x^3 - 3x^2 - \frac{15x}{4} = 0$

$x(4x^2 - 12x - 15) = 0$

x = 0

or $4x^2 - 12x - 15 = 0$

$$x = \frac{12 \pm \sqrt{144 + 240}}{8} = \frac{12 \pm \sqrt{384}}{8} = \frac{12 \pm 4\sqrt{24}}{8} = \frac{3 \pm \sqrt{24}}{2}$$

Reject $x = \frac{3 - \sqrt{24}}{2}$; hence $x = \frac{3 + \sqrt{24}}{2}$

$\therefore$ $x = \left\{0, \frac{7}{4}, \frac{3 + \sqrt{24}}{2}\right\}$]