VKR Classes

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B.Tech., IIT DELHI
with you since 15 years

JEE Advanced

Time : 2 hr.   Test Paper 2   Date 25/01/15   Batch - A   Marks : 120

SINGLE CORRECT CHOICE TYPE [3, –1]

1. Let f(x) = x + sin x. Suppose g denotes the inverse functions of f. The value of $g'\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)$ has the value equal to

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (C) $2-\sqrt{2}$ (D) $\sqrt{2}+1$

2. Let $f(x) = x^2 \ln(g(x))$ where g(x) is a differentiable positive function on (0, ∞) satisfying g(2) = 3, g'(2) = – 4, then f '(2) equals.

(A) $-\frac{16}{6}+8\ln(3)$ (B) $-\frac{16}{3}+4\ln(3)$ (C) $4\ln(3)$ (D) $-\frac{16}{3}+4\ln(6)$

3. Number of points on [0, 2] where $f(x) = \begin{cases} x\{x\}+1 & 0 \le x < 1 \\ 2-\{x\} & 1 \le x \le 2 \end{cases}$ fails to be continuous or derivable is

(A) 0 (B) 1 (C) 2 (D) 3

4. Given $f(x) = \begin{cases} \sqrt{10-x^2} & \text{if } -3 < x < 3 \\ 2-e^{x-3} & \text{if } x \ge 3 \end{cases}$

The graph of $f(x)$ is

(A) continuous and differentiable at x = 3 (B) continuous but not differentiable at x = 3
(C) differentiable but not continuous at x = 3 (D) neither differentiable nor continuous at x = 3

5. Let $f(x) = \lim_{n\to\infty} \frac{(x^2+2x+3+\sin\pi x)^n - 1}{(x^2+2x+3+\sin\pi x)^n + 1}$, then

(A) f(x) is continuous and differentiable for all x ∈ R.
(B) f(x) is continuous but not differentiable for all x ∈ R.
(C) f(x) is discontinuous at infinite number of points.
(D) f(x) is discontinuous at finite number of points.

6. If f : [–2a, 2a] → R is an odd function such that f(x) = f(2a – x) for x ∈ (a, 2a). if the left hand derivative of f(x) at x = a is zero, then the left hand derivative of f(x) at x = –a is

(A) 1 (B) –1 (C) 0 (D) none

7. If g′(x) exists for all x, g′ (0) = 2 and $g(x + y) = e^y g(x) + e^x g(y)$ ∀ x, y. Then

(A) $g(2x) = -2e^x g(x)$ (B) $g'(x) = g(x) + 2e^x$ (C) $\lim_{h\to 0} g(h)/h = 3$ (D) None of these

8. If $y = (1 - x)^{-\alpha} e^{-\alpha x}$ x ≠ 1 then

(A) $(1-x)y'' + (1-\alpha x)y' - \alpha y = 0$ (B) $(1-x)y'' - (1+\alpha x)y' - \alpha y = 0$
(C) $(1-x)y'' + (1+\alpha x)y' - \alpha y = 0$ (D) $(1-x)y'' + (1+\alpha x)y' + \alpha y = 0$

COMPREHENSION [3, -1]

Comprehension # 1

Consider the function $f : R \to R$, defined as $f(x) = \begin{cases} x^2 - x + 3, & x \in (-\infty, 3) \cap Q \\ x + a, & x \in (-\infty, 2) - Q \\ 2^x + 1, & x \in (2, 3) - Q \\ 9\tan\left(\dfrac{\pi x}{12}\right), & x \in [3, 6) \end{cases}$

9. If f(x) is continuous at x = 2 then the value of a is

(A) 1 (B) 2 (C) 3 (D) indeterminate

10. At x = 3 the function f(x)

(A) has a non-removable discontinuity (B) has a removable discontinuity

(C) is differentiable (D) is continuous but not differentiable

Comprehension # 2

Let f(x) is a function continuous for all $x \in R$ except at x = 0. Such that $f'(x) < 0 \ \forall\ x \in (-\infty, 0)$ and $f'(x) > 0 \ \forall\ x \in (0, \infty)$. Let $\lim\limits_{x \to 0^+} f(x) = 2$, $\lim\limits_{x \to 0^-} f(x) = 3$ and $f(0) = 4$.

11. The values of $\lim\limits_{x \to 0^+} \dfrac{f(-x)\, x^2}{\left\{\dfrac{1 - \cos x}{[f(x)]}\right\}}$ where [·] denote greatest integer function and { · } denote fraction part function.

(A) 6 (B) 12 (C) 18 (D) 24

12. $\lim\limits_{x \to 0^-} \left(\left[3f\left(\dfrac{x^3 - \sin^3 x}{x^4}\right)\right] - f\left(\left[\dfrac{\sin x^3}{x}\right]\right)\right)$ where [·] denote greatest integer function.

(A) 3 (B) 5 (C) 7 (D) 9

Comprehension # 3

In a certain problem the differentiation of product (f(x).g(x)) appears. One student commits mistake and differentiates as $\dfrac{df}{dx}.\dfrac{dg}{dx}$ but he gets correct result if $f(x) = x^3$ and g(4) = 9, g(2) = –9 and $g(0) = \dfrac{1}{3}$.

13. The derivative of f(x – 3) . g(x) with respect to x at x = 100 is

(A) 0 (B) 1 (C) –1 (D) 2

14. $\lim\limits_{x \to 0} \dfrac{f(x).g(x)}{x(1 + g(x))}$ will be

(A) 0 (B) –1 (C) 1 (D) 2

Comprehension # 4

Consider a differentiable function $f : R \to R$ which satisfies $f^2\left(\dfrac{1}{\sqrt{2}}x\right) = f(x)$ for all $x \in R$ and f(1) = 2.

15. If $\alpha, \beta \in R$ satisfying $\alpha^2 + \beta^2 = 1$, then for all x, $f(\alpha x) . f(\beta x) =$

(A) 2f(x) (B) f(x) (C) $\alpha\beta f^2(x)$ (D) None of these

16. $\int f(\sqrt{2}\,x) \,.\, \ln 2^{4x}$ is equal to

(A) $2^{x^2+1} + c$ (B) $2^{2x} + c$ (C) $2^{2x^2-1} + c$ (D) $2^{2x^2} + c$

MULTIPLE CORRECT CHOICE TYPE [3, 0]

17. Suppose J = $\int \frac{\sin^2 x + \sin x}{1 + \sin x + \cos x} dx$ and K = $\int \frac{\cos^2 x + \cos x}{1 + \sin x + \cos x} dx$. If C is an arbitrary constant of integration then which of the following is/are correct?

(A) J = $\frac{1}{2}$(x – sin x + cos x) + C

(B) J = K – (sin x + cos x) + C

(C) J = x – K + C

(D) K = $\frac{1}{2}$(x – sin x + cos x) + C

18. For a function f(x) = $\frac{ln(\{\sin x\} \cdot \{\cos x\} + 1)}{\{\sin x\}\{\cos x\}}$ where {x} denotes fractional part function then

(A) $f(0^-) = f\left(\frac{\pi}{2}^+\right)$

(B) $f\left(\frac{\pi}{2}^-\right) = f(0^+)$

(C) $\lim_{x \to 0} f(x) = 1$

(D) $\lim_{x \to \pi/2} f(x) = 1$

19. Let f (x) = $x^2 \sin \frac{1}{x}$ for 0 < x ≤ 1 and f (0) = 0. If g (x) = x^2 for x ∈ [0, 1] then which of the following statement(s) is/are correct?

(A) f (x) is differentiable in [0, 1]

(B) $\lim_{x \to 0} \frac{f(x)}{g(x)}$ does not exist.

(C) $\lim_{x \to 0} f'(x)g'(x)$ does not exist

(D) $\lim_{x \to 0} \frac{f'(x)}{g'(x)}$ does not exist.

20. Let y = $\sqrt{(\sin x + \sin 2x + \sin 3x)^2 + (\cos x + \cos 2x + \cos 3x)^2}$ then which of the following is correct?

(A) $\frac{dy}{dx}$ when x = $\frac{\pi}{2}$ is – 2

(B) value of y when x = $\frac{\pi}{5}$ is $\frac{3 + \sqrt{5}}{2}$

(C) value of y when x = $\frac{\pi}{12}$ is $\frac{\sqrt{1} + \sqrt{2} + \sqrt{3}}{2}$

(D) y simplifies to (1 + 2 cos x) in [0, π]

21. The functionf f(x) = max (|tan x|, cos|x|) is

(A) not differentiable at 4 points in (–π, π)

(B) discontinuous at 2 points in (–π, π)

(C) not differentiable at only 2 points in (–π, π)

(D) in (–π, π) there are onl 2 points where f(x) is continuous but not differentiable

22. Let a differentiable function f(x) be such that |f(y) – f(y)| ≤ $\frac{1}{2}$ |x – y| ∀ x, y ∈R and f'(x) ≥ $\frac{1}{2}$. Then the number of points of intersection of the graph of y = f(x) with

(A) the line y = x is one

(B) the curve y = $-x^3$ is one

(C) the curve 2y = |x| is three

(D) None of these

23. Let f (x)= | x – 1 | ([x] – [–x]) , then which of the following statement(s) is/are correct?
(where [x] denotes greatest integer function)

(A) f (x) is continuous at x = 1

(B) f (x) is derivable at x = 1

(C) f (x) is non-derivable at x = 1

(D) f (x) is discontinuous at x = 1.

24. Which of the following statement(s) is/are correct?

(A) Let f and g be defined on R and c be any real number. If $\lim_{x\to c} f(x) = b$ and g (x) is continuous at x = b then $\lim_{x\to c} g(f(x)) = g(b)$.

(B) There exist a function f : [0, 1] $\to$ R which is discontinuous at every point in [0, 1] and |f(x)| is continuous at every point in [0, 1].

(C) If f (x) and g (x) are two continuous function defined from R $\to$ R such that f (r) = g (r) for all rational numbers 'r' then f (x) = g (x) $\forall$ x $\in$ R.

(D) If f (a) and f (b) possesses opposite signs then there must exist atleast one solution of the equation f (x) = 0 in (a, b) provided f is continuous in [a, b].

25. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax-b\right)=0$, then for k $\geq$ 2, k $\in$ N which of the following is/are correct?

(A) 2a + b = 0 (B) a + 2b = 0

(C) $\lim_{n\to\infty} \sec^{2n}(k!\pi b)=1$ (D) $\lim_{n\to\infty} \sec^{2n}(k!\pi a)=1$

26. Which of the following functions are not derivable at x = 0?

(A) f (x) = $\sin^{-1} 2x\sqrt{1-x^2}$ (B) g (x) = $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

(C) h (x) = $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (D) k (x) = $\sin^{-1}(\cos x)$

INTEGER ANSWER TYPE [3, 0]

27. If $\int \frac{\sin^3 \frac{\theta}{2} d\theta}{\cos\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}} = \tan^{-1}\sqrt{f(\theta)} + c$ then the least value of f(θ) for allowable values of θ is equal to :

28. If the function *f* (x) = $\lim_{n\to\infty} \frac{x^n\left(a+\sin(x^n)\right)+\left(b-\sin(x^n)\right)}{(1+x^n)\sec\left(\tan^{-1}(x^n+x^{-n})\right)}$ is continuous at x = 1 then a + b is equal to

29. Given *f* ''(x) = cos x, $f'\left(\frac{3\pi}{2}\right)$ = e and *f* (0) = 1, then *f* (x) = (e + a)x – cos x + b where a + b is equal to

30. Let g(x) = f(x) sin x, where f(x) is a twice differentiable function on (– ∞, ∞) such that f '(–π) = 1. The value of | g''(– π) |equals

31. If the differential coefficients of $\sin^{-1}(2x\sqrt{1-x^2})$ at $x = -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}$ are a, b, c respectively, then find the value of a + b – c.

32. Let f(x) and g(x) are differentiable functions satisfying the conditions ;

(i) f (0) = 2 ; g (0) = 1

(ii) f ′ (x) = g (x)

and (iii) g ′ (x) = f (x).

Find the value of $(f(1))^2 - (g(1))^2$.

33. A function $f(x)$ continuous on R and periodic with period 2π satisfies $f(x) + \sin x \cdot f(x + \pi) = \sin^2 x$. The value of $(2f(\pi/6))^{-1}$

34. Let $f(x) = x^n$, $n \in W$. The number of values of n for which $f'(p + q) = f'(p) + f'(q)$ is valid for all positive p and q is

35. Let $f(x) = (x^2 - 4)\,|(x^3 - 6x^2 + 11x - 6)| + \frac{x}{1+|x|}$. Find the number of points at which the function f(x) is not differentiable.

36. If $f'(a) = \frac{1}{4}$, then $\lim_{h\to 0} \frac{f(a+2h^2) - f(a-2h^2)}{f(a+h^3-h^2) - f(a-h^3+h^2)}$ is equal to

<u>MATRIX MATCH TYPE</u> [3, –1]

37. Match the following

	Column-I		Column-II
(A)	If $\int x^2 \, d(\tan^{-1} x) = x f(x) + c$, then f(1) is equal to	(P)	0
(B)	If $\int \sqrt{1+3\tan x(\tan x + \sec x)}\,dx = a \log\left|\cos\frac{x}{2} - \sin\frac{x}{2}\right| + c$ then a is equal to $(0 < x < \frac{\pi}{2})$	(Q)	-2
(C)	If $\int x^2 e^{2x} dx = e^{2x} f(x) + c$, then the minimum value of f(x) is equal to	(R)	$\frac{\pi}{4}$
(D)	If $\int \frac{x^4+1}{x(x^2+1)^2} dx = a \log\lvert x\rvert + \frac{b}{x^2+1} + c$ then a – b is equal to	(S)	$\frac{1}{8}$

38.

	Column – I		Column – II
(A)	If $f(xy) = f(x) \cdot f(y)$ and f is differentiable at x = 1 such that $f'(1) = 1$ also $f(1) \neq 0$, then $f'(7)$ equals	(P)	1
(B)	If [.] denotes greatest integer fuction, then number of points at which the function $f(x) = \lvert x^2 - 3x + 2\rvert + \lvert\sin x\rvert - [x - 1/2]$, $-\pi \le x \le \pi$, is non differentiable, is	(Q)	13
(C)	Let $f(x) = [a + 7\sin x]$, $x \in (0, \pi)$, $a \in Z$ and [.] denotes greatest integer function. Then number of points at which f(x) is not differentiable is	(R)	5
(D)	Differential coefficient of $\sin^{-1}\frac{2x}{1+x^2}$ with respect to $\cos^{-1}\frac{1-x^2}{1+x^2}$ in the domain of $f(x) = \frac{1}{\sqrt{1-x^2}}$ is	(S)	– 1

39. **Column – I** **Column – II**

(A) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin(\cos x)}{\cot x}$ equals — (P) 1

(B) $\left(\lim\limits_{x \to 0} \frac{\sin 3x - \sin x}{\ln(x+1)}\right)^{-1}$ equals — (Q) $\lim\limits_{x \to 0} \frac{1-\sqrt{1-4x^2}}{x^2}$

(C) $\lim\limits_{x \to 0} \frac{25}{3}\left(\frac{\sqrt[5]{(1+x)^3}-1}{(1+x)\sqrt[3]{(1+x)^2}-1}\right)$ equals — (R) $\frac{1}{2}$

(D) $\lim\limits_{x \to 0} \frac{\ln x + \ln\left(\frac{1}{x} + 3\sin x\right)}{\tan(x^2)}$ equals — (S) 3

40. **Column - I** **Column - II**

(A) Number of natural numbers less than the fundamental period of $\sin^2 x + \sec^2 x - \tan^2 x$ — (P) 1

(B) Number of points of discontinuity of the function f(x) = [x] + {2x} + [3x] for x $\in$ [0, 1], where [.] and { . } represent greatest integer and fractional part functions respectively. — (Q) 2

(C) $\left[\lim\limits_{x \to 0} \frac{\sin x(1+\cos x)}{x \cos x}\right]$, where [.] represents greatest integer function — (R) 3

(D) Number of solution of the equation $\sin^{-1} x - 2\cos^{-1}(1 + x) = 0$. — (S) 4

Answer Sheet

Student Name: ______________________________ **Batch : A** **Date : 25/01/15**

1. Ⓐ Ⓑ Ⓒ Ⓓ	2. Ⓐ Ⓑ Ⓒ Ⓓ	3. Ⓐ Ⓑ Ⓒ Ⓓ	4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ	6. Ⓐ Ⓑ Ⓒ Ⓓ	7. Ⓐ Ⓑ Ⓒ Ⓓ	8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ	10. Ⓐ Ⓑ Ⓒ Ⓓ	11. Ⓐ Ⓑ Ⓒ Ⓓ	12. Ⓐ Ⓑ Ⓒ Ⓓ
13. Ⓐ Ⓑ Ⓒ Ⓓ	14. Ⓐ Ⓑ Ⓒ Ⓓ	15. Ⓐ Ⓑ Ⓒ Ⓓ	16. Ⓐ Ⓑ Ⓒ Ⓓ
17. Ⓐ Ⓑ Ⓒ Ⓓ	18. Ⓐ Ⓑ Ⓒ Ⓓ	19. Ⓐ Ⓑ Ⓒ Ⓓ	20. Ⓐ Ⓑ Ⓒ Ⓓ
21. Ⓐ Ⓑ Ⓒ Ⓓ	22. Ⓐ Ⓑ Ⓒ Ⓓ	23. Ⓐ Ⓑ Ⓒ Ⓓ	24. Ⓐ Ⓑ Ⓒ Ⓓ
25. Ⓐ Ⓑ Ⓒ Ⓓ	26. Ⓐ Ⓑ Ⓒ Ⓓ		

27. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	28. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
29. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	30. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
31. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	32. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
33. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	34. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
35. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	36. ⓪ ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

37. A	B	C	D	38. A	B	C	D	39. A	B	C	D	40. A	B	C	D
Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ	Ⓟ
Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ	Ⓠ
Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ	Ⓡ
Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ	Ⓢ

JEE Advanced

Test Paper 2 **Batch - A** **Date 25/01/15**

ANSWER WITH SOLUTION

ANSWER KEY

Q.	1	2	3	4	5	6	7	8	9	10	11	12
A.	C	B	C	B	A	C	B	B	C	D	B	B
Q.	13	14	15	16	17	18	19	20	21	22	23	24
A.	A	A	B	D	BC	AB	ABD	AB	ABD	AB	AC	All
Q.	25	26	27	28	29	30	31	32	33	34	35	36
A.	BCD	BCD	3	0	3	2	2	3	5	2	2	0
Q.	37.						38.					
A.	(A) – (R), B – (Q), (C) – (S), (D) – (P)						(A) – (P), B – (R), (C) – (Q), (D) – (S)					
Q.	39.						40.					
A.	(A) – (P), B – (Q,R), (C) – (S), (D) – (S)						(A) – (R), B – (S), (C) – (Q), (D) – (P)					

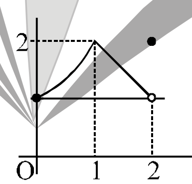
SOLUTION

2. $f'(x) = \frac{x^2}{g(x)} \cdot g'(x) + \ln g(x) \cdot 2x$

$f'(2) = \frac{4}{3}(-4) + \ln(3) \cdot 4$ = 4 $\ln$ (3) – $\frac{16}{3}$ Ans.]

3. x = 1 continuous but not derivable
x = 2 discontinuous and not derivable

$$f(x) = \begin{cases} x^2+1 & 0 \le x < 1 \\ 3-x & 1 \le x < 2 \\ 2 & x = 2 \end{cases}$$



]

4. $f'(3^+) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(2-e^h)-1}{h} = -\lim_{h\to 0}\left(\frac{e^h-1}{h}\right) = -1$

$f'(3^-) = \lim_{h\to 0} \frac{f(3-h)-f(3)}{-h} = \lim_{h\to 0} \frac{\sqrt{10-(3-h)^2}-1}{-h} = \lim_{h\to 0} \frac{\sqrt{1+(6h-h^2)}-1}{-h}$

$= \lim_{h\to 0} \frac{(6h-h^2)}{-h(\sqrt{1+6h-h^2}+1)} = \lim_{h\to 0} \frac{h(h-6)}{h(\sqrt{1+6h-h^2}+1)} = \frac{-6}{2} = -3$

Hence $f'(3^+) \neq f'(3^-) \Rightarrow$ (B)]

11. $\lim_{x\to 0^+} \frac{f(-x)x^2}{\left(\frac{1-\cos x}{[f(x)]}\right) - \left[\frac{1-\cos x}{[f(x)]}\right]} = \frac{3x^2}{\frac{1-\cos x}{2} - 0}$

= 6 × 2 = 12 **Ans.**

12. $x \to 0^{-}$ $\left(\frac{x^3 - \sin^3 x}{x^4}\right) = \left(\frac{x - \sin x}{x^3}\right)\left(\frac{x^2 + \sin^2 x + x \sin x}{x^2}\right) x = \frac{1}{6}$ (3) $x \to 0^{-} \Rightarrow f(0^{-}) = 3$

$x \to 0^{-}$ $\frac{\sin x^3}{x} = \frac{\sin x^3}{x^3} x^2 \to 0^{+}$ $\Rightarrow \left[\frac{\sin x^3}{x}\right] = 0$ $\Rightarrow$ $f(0) = 4$

$\therefore$ $3f\left(\frac{x^3 - \sin^3 x}{x^4}\right) > 9$

$\Rightarrow$ $[9^{+}] - f(0) = 9 - 4 = 5$ **Ans.**]

15. $f^2\left(\frac{1}{\sqrt{2}} x\right) = f(x)$

Differentiate both sides w.r.t. x $\sqrt{2}\, f\left(\frac{1}{\sqrt{2}x}\right) f'\left(\frac{1}{\sqrt{2}} x\right) = f'(x)$

$$\frac{\frac{x}{\sqrt{2}} f\left(\frac{1}{\sqrt{2}} x\right)}{f'\left(\frac{1}{\sqrt{2}} x\right)} = \frac{x f(x)}{f'(x)} \Rightarrow \frac{x f(x)}{f'(x)} = \text{constant} \Rightarrow \frac{f'(x)}{f(x)} = c\,x$$

$\ell n\,|f(x)| = \frac{cx^2}{2} + k$ $\quad f(x) = A\,.\,e^{\frac{cx^2}{2}}$

$f(0) = 1,\ f(1) = 2 \Rightarrow f(x) = 2^{x^2}$

$f(\alpha x) + f(\beta x) = 2^{(\alpha^2 + \beta^2)x^2} = 2^{x^2} = f(x).$

16. $\int 2^{(\sqrt{2}x)^2}\, \ell n\, 2^{4x}\, dx = 2\, \ell n\, 2 \int 2x . 2^{2x^2}\, dx$

Put $x^2 = t$

$= 2\, \ell n\, 2 \int 2^{2t}\, dt = 2\, \ell n\, 2\, .\, \frac{2^{2t}}{2\,\ell n 2} + c = 2^{2t} + c = 2^{2x^2} + c$

17. $J + K = \int \frac{1 + \sin x + \cos x}{1 + \sin x + \cos x} dx$

$J + K = x + C$ ....(1) $\Rightarrow$ **(C)**

again $J - K = \int \frac{(\sin^2 x - \cos^2 x) + \sin x - \cos x}{1 + \sin x + \cos x} dx = \int \frac{(\sin x - \cos x) + (\sin x + \cos x + 1)}{1 + \sin x + \cos x} dx$

$J - K = -\cos x - \sin x + C$ ....(2)

hence $J = K - (\sin x + \cos x) + C$ $\Rightarrow$ **(B)**

Also (1) + (2)

$2J = x - (\cos x + \sin x) + C$

$J = \frac{1}{2}[x - \sin x - \cos x] + C$

and (1) – (2)
$2K = x + (\sin x + \cos x) + C$

$K = \frac{1}{2}(x + \sin x + \cos x) + C$

from (1) $J = x - K + C \Rightarrow$ **(C)**]

18. $f(0^+) = \lim_{h\to 0} \frac{ln(\sin h \cdot \cos h + 1)}{\sin h \cdot \cos h} = \lim_{h\to 0} \frac{ln(\sin h \cdot \cos h + 1)}{\frac{\sin h}{h} \cdot h \cos h}$

$= \lim_{h\to 0} ln(\sin h \cdot \cos h + 1)^{1/h} = \lim_{h\to 0} \frac{1}{h}(\sin h \cdot \cos h + 1 - 1) = 1$

$f\left(\frac{\pi}{2}^-\right) = \lim_{h\to 0} \frac{ln(\cos h \cdot \sin h + 1)}{\cos h \cdot \sin h} = f(0^+) \Rightarrow$ **(B) Ans.**

$f(0^-) = \lim_{h\to 0} \frac{ln((1-\sin h)\cos h + 1)}{(1-\sin h)\cos h}$

$f(0^-) = ln\,2 \Rightarrow \lim_{x\to 0} f(x)$ does not exist

$f\left(\frac{\pi}{2}^+\right)$ is same as $f(0^-) \Rightarrow$ **(A) Ans.** and $\lim_{x\to \pi/2} f(x)$ does not exist]

19. $f'(0) = 0$; $f'(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x} & \text{for } 0 < x \le 1 \\ 0 \text{ if } x = 0 \end{cases}$

$\Rightarrow$ f(x) is differentiable in [0, 1]

$\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{x^2\sin(1/x)}{x^2}$ D.N.E. $\Rightarrow$ **(B)**

$\lim_{x\to 0} f'(x)\,g'(x) = \lim_{x\to 0} 4x^2\sin\frac{1}{x} - 2x\cos\frac{1}{x} = 0 \Rightarrow$ **(C)** is not correct.

$\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ does not exist $\Rightarrow$ **(D)**]

20. $\sqrt{3 + 2(\cos x + \cos x + \cos 2x)} = \sqrt{3 + 2(2\cos x + 2\cos^2 x - 1)}$

$= \sqrt{4\cos^2 x + 4\cos x + 1} = \sqrt{(1 + 2\cos x)^2} = |1 + 2\cos x|$

$y = \begin{cases} 1 + 2\cos x & \text{if } x \in [0, 2\pi/3) \\ -(1 + 2\cos x) & \text{if } x \in [2\pi/3, \pi] \end{cases}$

now verify all the alternatives.]

25. $a = 1$; $b = -\frac{1}{2}$; (C) and (D) are obvious]

26. (A) not derivable at $x = \frac{1}{\sqrt{2}}$ check with $x = \sin\theta$ or direct diff.

(B) $g'(x) = \dfrac{1}{\sqrt{1-\left(\dfrac{2\cdot 2^x}{1+2^{2x}}\right)^2}} \cdot \dfrac{(1+4^x)(2^{x+1})\ln 2 - 2^{x+1}\cdot 4^x \cdot \ln 4}{(1+4^x)^2}$

$= \dfrac{(1+2^{2x})^2}{\sqrt{(1+2^{2x})^2-(2\cdot 2^x)^2}} = \dfrac{1}{|1-2^{2x}|} \Rightarrow$ not derivable at x = 0]

27. 3

$$I = \int \frac{\sin^2\frac{\theta}{2}}{\cos\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}}d\theta = \int \frac{\left(2\sin\frac{\theta}{2}\,.\,\cos\frac{\theta}{2}\right)\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}}d\theta$$

$$= \int \frac{2\sin^2\frac{\theta}{2}\sin\theta\,d\theta}{2\cos^2\frac{\theta}{2}\sqrt{\cos^3\theta+\cos^2\theta+\cos\theta}}$$

Put $\cos\theta = t \Rightarrow -\sin\theta\, d\theta = dt$

Also $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2} = t$

$$\therefore \quad I = \int \frac{\frac{1-t}{2}(-dt)}{(1+t)\sqrt{t^3+t^2+t}} = \frac{1}{2}\int \frac{(t^2-1)dt}{(t+1)^2\sqrt{t^3+t^2+t}} = \frac{1}{2}\int \frac{\left(1-\frac{1}{t^2}\right)(dt)}{\left(t+\frac{1}{t}+2\sqrt{t+\frac{1}{t}+1}\right)}$$

Put $t + \frac{1}{t} + 1 = u^2 \Rightarrow \left(1-\frac{1}{t^2}\right)dt = 2u\,du$

$$I = \frac{1}{2}\int \frac{2udu}{(1+u^2)u} = \tan^{-1}u = \tan^{-1}\sqrt{1+\frac{1}{t}+1} + c = \tan^{-1}(\cos\theta+\sec\theta+1)^{1/2} + c$$

So, $f(\theta) = \cos\theta + \sec\theta + 1 \geq 2 + 1 = 3$

28. 0

$$f(x) = \lim_{n\to\infty} \frac{x^n\left(a+\sin(x^n)\right)+\left(b-\sin(x^n)\right)}{(1+x^n)\sec\left(\tan^{-1}(x^n+x^{-n})\right)}$$

for continuity at x = 1

$\lim_{x\to 1} f(x)$ must exist and equals f (1)

$$f(1) = \lim_{n\to\infty}\frac{1^n(a+\sin 1^n)+b-\sin(1^n)}{(1+1^n)\cdot\sec\left(\tan^{-1}(1^n+1^{-n})\right)} = \frac{a+\sin 1+b-\sin 1}{\sec(\tan^{-1}2)} = \frac{a+b}{2\sqrt{5}}$$

Now for x > 1 in the immediate neighbourhood

$$f(x) = \lim_{n\to\infty}\frac{a+\sin(x^n)+\dfrac{b-\sin x^n}{x^n}}{\left(1+\dfrac{1}{x^n}\right)\sec\left(\tan^{-1}(x^n+x^{-n})\right)} = \frac{a+(\text{some quantity between 1 and }-1)+0}{1\cdot\sec(\tan^{-1}\infty)} = 0$$

|||ly for x < 0 in the immediate neighbourhood = 0

$$f(x) = \frac{b}{1\cdot\sec(\tan^{-1}\infty)} = 0$$

Hence f (x) = 0 for $x \neq 1$ $\therefore$ $\lim_{x\to1} f(x) = 0 = a + b$]

29. 3

$f''(x) = \cos x$

$f'(x) = \sin x + C$

$e = -1 + C \quad \Rightarrow \quad C = e + 1$

$f(x) = -\cos x + (e + 1)x + C_2$

$1 = -1 + C_2 \quad \Rightarrow \quad C_2 = 2 \quad \Rightarrow f(x) = (e + 1)x - \cos x + 2$

30. 2

We have g(x) = f(x) sin x ....(1)

On differentiating equation (1) w.r.t. x, we get

$g'(x) = f(x)\cos x + f'(x)\sin x$ ....(2)

Again differentiating equation (2) w.r.t. x, we get

$g''(x) = f(x)(-\sin x) + f'(x)\cos x + f'(x)\cos x + f''(x)\sin x$ ....(3)

$\Rightarrow \quad g''(-\pi) = 2f'(-\pi)\cos(-\pi) = 2 \times 1 \times -1 = -2$

Hence $g''(-\pi) = -2$

32. Ans. 3

$f'(x) = g(x) \qquad f(0) = 2$

$g'(x) = f(x) \qquad g(0) = 1$

add $\quad f'(x) + g'(x) = f(x) + g(x)$

$$\Rightarrow \quad \frac{f'(x)+g'(x)}{f(x)+g(x)} = 1 \quad \Rightarrow \quad ln(f(x) + g(x)) = x + c$$

Put x = 0 to get c = *l*n 3

hence $f(x) + g(x) = 3e^x$ ....(1)

similarly subtraction gives

$$\frac{f'(x)+g'(x)}{f(x)+g(x)} = -x$$

Integrating ln (f(x) – g(x)) = –x + c

c = *l*n(1) hence c = 0 $\Rightarrow$ $f(x) - g(x) = e^{-x}$ ....(2)

33. Ans. 5

$f(x) + \sin x \cdot f(x + \pi) = \sin^2 x$ ....(1)

$x \to x + \pi \qquad f(x + \pi) + \sin(\pi + x)\cdot f(x + 2\pi) = \sin^2(\pi + x)$

$\qquad f(x + \pi) - \sin x\, f(x) = \sin^2 x$ ....(2)

from (1) $\qquad -f(x + \pi) = \dfrac{\sin^2 x - f(x)}{\sin x}$

from (2) $\qquad f(x + \pi) = \sin^2 x + \sin x \cdot f(x)$

$$\therefore \quad \sin^2 x + \sin x \cdot f(x) = \frac{\sin^2 x - f(x)}{\sin x}$$

$\sin^3 x + \sin^2 x \cdot f(x) = \sin^2 x - f(x)$

$f(x)[1 + \sin^2 x] = \sin^2 x(1 - \sin x)$

$$\therefore \quad f(x) = \frac{\sin^2 x(1-\sin x)}{1+\sin^2 x}$$