VKR Classes

TIME BOUND TESTS 1-10

Target JEE ADVANCED

For Class XII

PRACTICE TEST-1

Comprehension Type

Paragraph for Question Nos. 1 to 3

Let $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ be two planes, where $d_1, d_2 > 0$. Then origin 'O' lies in acute angle if $a_1 a_2 + b_1 b_2 + c_1 c_2 < 0$ and origin lies in obtuse angle if $a_1 a_2 + b_1 b_2 + c_1 c_2 > 0$. Further point (x_1, y_1, z_1) and origin both lie either in acute angle or in obtuse angle, if $(a_1 x_1 + b_1 y_1 + c_1 z_1 + d_1)(a_2 x_1 + b_2 y_1 + c_2 z_1 + d_2) > 0$. One of (x_1, y_1, z_1) and origin lie in acute angle and the other in obtuse angle, if $(a_1 x_1 + b_1 y_1 + c_1 z_1 + d_1)(a_2 x_1 + b_2 y_1 + c_2 z_1 + d_2) < 0$.

1. Given the planes $2x + 3y - 4z + 7 = 0$ and $x - 2y + 3z - 5 = 0$, if a point P is $(1, -2, 3)$, then
(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle
(C) O lies in acute angle, P lies in obtuse angle
(D) O lies in obtuse angle, P lies in acute angle.

2. Given the planes $x + 2y - 3z + 5 = 0$ and $2x + y + 3z + 1 = 0$. If a point P is $(2, -1, 2)$, then
(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle
(C) O lies in acute angle, P lies in obtuse angle
(D) O lies in obtuse angle, P lies in acute angle.

3. Given the planes $x + 2y - 3z + 2 = 0$ and $x - 2y + 3z + 7 = 0$. If a point P is $(1, 2, 2)$, then
(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle
(C) O lies in acute angle, P lies in obtuse angle
(D) O lies in obtuse angle, P lies in acute angle.

Paragraph for Question Nos. 4 to 6

Consider a triangle ABC, where x, y, z are the length of perpendicular drawn from the vertices of the triangle to the opposite sides a, b, c respectively let the letters R, r, s, Δ denote the circumradius, inradius semiperimeter and area of the triangle respectively.

4. If $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b} = \frac{a^2 + b^2 + c^2}{k}$, then the value of k is

(A) R (B) s (C) 2R (D) $\frac{3}{2}$ R

5. If $\cot A + \cot B + \cot C = k\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$, then the value of k is

(A) R^2 (B) rR (C) Δ (D) $a^2 + b^2 + c^2$

6. The value of $\frac{c\sin B + b\sin C}{x} + \frac{a\sin C + c\sin A}{y} + \frac{b\sin A + a\sin B}{z}$ is equal to

(A) $\frac{R}{r}$ (B) $\frac{s}{R}$ (C) 2 (D) 6

Match the Column :

7. **Column-I** **Column-I**

(A) The equation of tangent of the ellipse which cuts off equal intercepts on axes is $x - y = a$ where a equal to — (P) 2

(B) The normal $y = mx - 2am - am^3$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex if m equals to — (Q) $\sqrt{3}$

(C) The equation of the common tangent to parabola $y^2 = 4x$ and $x^2 = 4y$ is $x + y + \frac{k}{\sqrt{3}} = 0$, then k is equal to — (R) $\sqrt{8}$

(D) An equation of common tangent to parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is $4x - 2y + \frac{k}{\sqrt{2}} = 0$, then k is equal to — (S) $\sqrt{41}$

8. **Column-I** **Column-I**

(A) In a ΔABC, let $\angle C = \frac{\pi}{2}$, r = inradius R = circumradius then $2(r + R)$ — (P) $a + b + c$

(B) It ℓ, m, n are perpendicular drawn from the vertices of triangle ΔABC teo sides a, b and c respectibvely then $\sqrt{2R\left(\frac{b\ell}{c}+\frac{cm}{a}+\frac{an}{b}\right)+2ab+2bc+2ca}$ — (Q) $a - b$

(C) In a ΔABC, $R(b^2 \sin 2C + c^2 \sin 2B)$ equals — (R) $a + b$

(D) In a right angle triangle ABC, $\angle C = \frac{\pi}{2}$, then $4R \sin \frac{A+B}{2}$. sub $\frac{(A-B)}{2}$ — (S) abc

Single Choice Question :

9. The number of integral solutions of $x_1 + x_2 + x_3 + x_4 \le 10$ where $x_1 \ge 2$, and x_2, x_3, x_4 are ≥ 0 is

(A) 505 (B) 715 (C) 495 (C) 792

10. Tthe value of $\frac{1.2}{3!}+\frac{2.2^2}{4!}+\frac{3.2^3}{5!}+......+\frac{15.2^{15}}{17!}$ equals

(A) $1 - \frac{2^{16}}{(17!)}$ (B) $1 - \frac{16.2^{17}}{(17!)}$ (C) $2 - \frac{2^{17}}{(17!)}$ (D) $2 - \frac{16.2^{17}}{(17!)}$

11. If $a_i^2 + b_i^2 + c_i^2 = 10$, i = 1, 2, 3 and $a_i a_j + b_i b_j + c_i c_j = 0$, $i \ne j$, i, j = 1, 2, 3 then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is

(A) 0 (B) 10 (C) 100 (D) 1000

12. If s_1, s_2, s_3 ...... s_n are the sums of infinite geometric series whose first terms are 1, 3, 5,......(2n - 1) and whose common ratios are $\frac{2}{3}, \frac{2}{5}, ...\frac{2}{2n+1}$ respectively, then $\lim_{n\to\infty}\left\{\frac{1}{s_1s_2s_3}+\frac{1}{s_2s_3s_4}+....\frac{1}{s_ns_{n+1}s_{n+2}}\right\} =$

(A) $\frac{1}{15}$ (B) $\frac{1}{60}$ (C) $\frac{1}{12}$ (D) $\frac{1}{3}$

13. If 3^k is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^{(n+1)}C_1 & {}^{(n+4)}C_1 & {}^{(n+7)}C_1 \\ {}^{(n+1)}C_2 & {}^{(n+4)}C_2 & {}^{(n+7)}C_2 \end{vmatrix}$, then the maximum value of k is

(A) 4 (B) 3 (C) 2 (D) 6

14. Let $S_r = \alpha^r + \beta^r + \gamma^r$. Then $D = \begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix}$ is equal to

(A) $(\beta-\alpha)(\gamma-\beta)(\alpha-\gamma)$ (B) $\{(\beta-\alpha)(\gamma-\beta)(\alpha-\gamma)\}^2$
(C) $(\beta+\alpha)(\gamma+\beta)(\alpha+\gamma)$ (D) 0

One or More than One Correct :

15. The value of $\sum_{r=0}^{n}(-2)^r \frac{{}^nC_r}{{}^{(r+2)}C_r}$ is equal to

(A) $\frac{1}{n+1}$ if n is even (B) $\frac{1}{n+2}$ if n is odd (C) $\frac{1}{n+2}$ if n is even (D) $\frac{1}{(n+1)(n+2)}$

16. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, then which of the following is/are possible

(A) $\frac{x}{y}+\frac{y}{x} = 2\cos(\alpha-\beta)$ (B) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

(C) $\frac{y^m}{y^n}+\frac{y^n}{x^m} = 2\cos(m\alpha + n\beta)$ (D) $xy + \frac{1}{xy} = 2\cos(\alpha+\beta)$

17. If $\alpha + i\beta = (\sqrt{3} + i)^{3n} \times (1-i)^{-2n_2}$, $n_1, n_2 \in N$, then
(A) $\alpha = 0$ if both n_1 and n_2 are even
(B) $\beta = 0$ if both n_1 and n_2 are odd
(C) $\alpha = 0$ if both one of n_1 and n_2 is odd
(D) $|\alpha + i\beta|$ is an even number provided $n_2 \neq 3n_1$

18. A variable plane passes through a fixed point (a, b, c) and meets the coordinates axes in A, B, C. The locus of the point comon to the planes through A, B, C parallel to the coordinats planes is
(A) $ayz + bzx + cxy = xyz$ (B) $ayz + bzx + cxy = 2xyz$

(C) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$ (D) $ax + by + cz = 1$

PRACTICE TEST-2

Single Choice Question :

1. $\int_0^1 \frac{x^6 - x^3}{(2x^3+1)^3}\, dx =$

(A) 0 (B) – 1/6 (C) – 1/18 (D) – 1/36

2. If f(x + f(y)) = f(x) + y $\forall$ x, y $\in$ R and f(0) = 1, then $\int_0^{10} f(10-x)$ dx is equal to

(A) 1 (B) 10 (C) $\int_0^1 f(x)$ dx (D) 10 $\int_0^1 f(x)$ dx

3. let g(x) = $e^{f(x)}$ and f(x + 1) = x + f(x), then X = $\frac{g'\left(N+\frac{1}{2}\right)g'\left(\frac{1}{2}\right)}{g\left(N+\frac{1}{2}\right)g\left(\frac{1}{2}\right)}$ is equal to (N $\in$ natural number)

(A) X = 2 $\left(1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{N}\right)$ (B) X = 2 $\left(1+\frac{1}{3}+\frac{1}{7}+......\frac{1}{2N-1}\right)$

(C) X = 0 (D) none of these

4. In a circle with centre O, OA, OB are radii $\angle AOB = 90°$. A semi-circle (S_1) is constructed using segment AB as its diameter non-overlapping with ΔOAB. The ratio of the area of S_1 outside given circle to the area of ΔOAB is equal to

(A) 1/2 (B) $\pi/4$ (C) 1 (D) none of these

5. If f(x) is a continuous function in [0, π] such that f(0) = f(π) = 0 then the value of

$\int_0^{\pi/2} (f(2x)+f''(2x))$ sin x . cos x dx is equal to

(A) π (B) 2π (C) 3π (D) none of these

6. Let f(x), x $\in$ [0, ∞) be a non-negative continuous function. If f'(x) cos x $\le$ f(x) sin x $\forall$ x $\ge$ 0 then the value of f(2π) is equal to

(A) 0 (B) 1 (C) π (D) none of these

One or more than one correct

7. If f''(sin x) > 0 $\forall$ x $\in$ R then which of the following are always correct for g(x) = f'(sin x)

(A) g is monotonically strictly increasing $\forall$ x $\in$ R

(B) g is monotonically strictly increasing $\forall$ x $\in$ $\bigcup_{n\in I}\left(2n\pi-\frac{\pi}{2}, 2n\pi+\frac{\pi}{2}\right)$

(C) g is monotonically strictly decreasing $\forall$ x $\in$ $\bigcup_{n\in I}(n\pi, (n+1)\pi)$

(D) g is monotonically strictly decreasing $\forall$ x $\in$ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

8. If y = f(x) has finite slope everywhere is an invertible function, then

(A) y = f(x) and $y = f^{-1}(x)$ cannot intersect orthogonally on line y = x

(B) y = f(x) and $y = f^{-1}(x)$ may have points of intersection not on line y = x

(C) If $f(x) \neq x$ then $\int_a^b f(x)\,dx \neq \int_a^b f^{-1}(x)\,dx$ $(a \neq b)$

(D) $\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx = b\,f(b) - a\,f(a)$, Given that $f'(x) > 0 \;\forall\; x \in R$

9. $T_n = \sum_{r=2n}^{3n-1} \frac{rn}{r^2 + n^2}$, $S_n = \sum_{r=2n+1}^{3n} \frac{rn}{r^2 + n^2}$, then $\forall\; n \in \{1, 2, 3......\}$

(A) $T_n > \frac{1}{2} \ln 2$ (B) $S_n < \frac{1}{2} \ln 2$ (C) $T_n < \frac{1}{2} \ln 2$ (D) $S_n > \frac{1}{2} \ln 2$

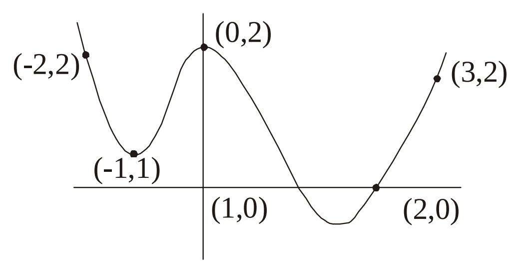
10. A line x = k intersects the graph of $y = \log_5 x$ and the graph of $y = \log_5 (x + 4)$. The distance between the points of intersection is 0.5. Given that $k = a + \sqrt{b}$ where $a, b \in N$ then

(A) a = 2, b = 4 (B) a = 1, b = 5 (C) a + b = 6 (D) a = b = 3

Comprehension Type

Paragraph for Question Nos. 11 to 13

In the given figure graph of



$y = p(x) = x^4 + ax^3 + bx^2 + cx + d$ is given

11. The product of all imaginary roots of p(x) = 0

(A) – 2 (B) – 1 (C) – 1/2 (D) none of these

12. If p(x) + k = 0 has 4 distinct real roots $\alpha, \beta, \gamma, \delta$ then $[\alpha] + [\beta] + [\gamma] + [\delta]$, (where [.] denotes greatest integer function) is euqal to

(A) – 1 (B) – 2 (C) 0 (D) 1

13. The minimum number of real roots of equation $(p'(x))^2 + p(x)\,p''(x) = 0$ are

(A) 3 (B) 4 (C) 5 (D) 6

Paragraph for Question Nos. 14 to 16

Given f is a twice differentiable function $\forall\; x \in R$ and $f'(x) \neq 0$. If $f(x) + f''(x) = -xh(x)\,f'(x)$ where $h(x) > 0 \;\forall\; x \in R$, $f(0) = -3$, $f'(0) = 4$ and $F(x) = (f(x))^2 + (f'(x))^2$ then

14. F(x) is monotonically strictly decreasing function in

(A) (–5, 5) (B) (–4, 4) (C) $(-\infty, 0)$ (D) $(0, \infty)$

15. If $|F(x)| \leq M \;\forall\; x \in (0, \infty)$ then least possible value of such M is

(A) 4 (B) 9 (C) 16 (D) 25

16. The number of points of extrema for F(x) is/are

(A) 0 (B) 1 (C) 2 (D) infinite

Match the Column

17. $f(x) = |9 - x^2| - |x - a|$ then

	Column-I		**Column-II**
(A)	For $a = 8$, $f(x) = 0$ has	(P)	exactly 4 distinct real roots
(B)	For $a = 3$, $f(x) = 0$ has	(Q)	exactly 3 distinct real roots
(C)	For $a = 10$, $f(x) = 0$ has	(R)	exactly 2 distinct real roots
(D)	For $a = -2$, $f(x) = 0$ has	(S)	exactly one distinct real roots

18.

	Column-I		**Column-II**
(A)	$f(x) = \dfrac{\tan^2 x - \tan x + 1}{\tan^2 x + \tan x + 1}$ on $x \in \left[0, \dfrac{\pi}{2}\right)$	(P)	greatest value of f = 1
(B)	$f(x) = \dfrac{2}{\pi}(\sin 2x - x)$ on $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$	(Q)	least value of f = – 1
(C)	$f(x) = 2x^3 - 3x^2$ on $x \in \left[\dfrac{1}{2}, \dfrac{3}{2}\right]$	(R)	maximum value of f = – 1
(D)	$f(x) = x \sin x + \cos x - \dfrac{1}{2}x^2$ on $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$	(S)	minimum value of f = 1

PRACTICE TEST-3

Single Choice Question :

1. A circle C_1 is drawn having any pint P on x-axis as its centre and passing through the centre of the circle(C) $x^2 + y^2 = 1$. A common tangent to C_1 and C intersects the circles at Q and R respectively. Then, Q(x, y) always satisfies

(A) $x^2 - 1 = 0$ (B) $x^2 + y^2 = 1$ (C) $y^2 - 1 = 0$ (D) None of these

2. If $y = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + ... + \infty$, then tan y is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) None of these

3. The minimum distance between the circle $x^2 + y^2 = 9$ and the curve $2x^2 + 10y^2 + 6xy = 1$ is

(A) $2\sqrt{2}$ (B) 2 (C) $3 - \sqrt{2}$ (D) $3 - \frac{1}{\sqrt{11}}$

4. The medians AA' and BB' of triangle ABC intersect at right angle. If BC = 3, AC = 4, then AB is

(A) $\sqrt{5}$ (B) 5 (C) $\sqrt{3}$ (D) None of these

One or more tan one correct :

5. If $25a^2 + 16b^2 - 40ab - c^2 = 0$, then the family of straight line $2ax + by + c = 0$ is current at

(A) $\left(-\frac{5}{2}, 4\right)$ (B) $\left(\frac{5}{2}, -4\right)$ (C) $\left(-\frac{5}{2}, -4\right)$ (D) $\left(\frac{5}{2}, 4\right)$

6. Let f be a differential function such that $f(x) = f(2 - x)\ \forall\ x \in R$ and $g(x) = f(1 + x)$ then

(A) g(x) is an odd function (B) g(x) is an even function

(C) graph of f(x) is symmetrical w.r.t. line x=1 (D) $f'(1) = 0$

7. A sequence is defined as $a_1 = a_2 = 1$ and $a_n = a_{n+1} + a_{n-2}\ \forall\ n \geq 3$ then

(A) $\frac{1}{a_{n-1}a_{n+1}} = \frac{1}{a_{n-1}a_n} - \frac{1}{a_n a_{n+1}}$ (B) $\sum_{n=2}^{\infty} \frac{1}{a_{n-1}a_{n+1}} = 1$

(C) $\sum_{n=2}^{\infty} \frac{a_n}{a_{n-1}a_{n+1}} = 2$ (D) $\{a_n\}$ is an increasing sequence

8. $f : R^+ \to R^+$ be a differentiable function such that $f'\left(\frac{a}{x}\right) = \frac{x}{f(x)}$, $\forall\ x \in R^+$ also $a \in R^+$ and $f(1)=f'(1)=1$ then

(A) $\frac{x}{f(x)} = \frac{f'(x) + xf''(x)}{(f'(x))^2}$ (B) $f(x) = xf'(x)$

(C) $f(x) = x$ (D) $f(x) = x^2$

Match the Column :

9. Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8} and $f : A \to B$, then number of functions f possible or

	Column - I		Column - II
(A)	$i + f(i) < 10,\ \forall\ i = \{1, 2, 3, 4\}$	(P)	128
(B)	$f(i) - 1 > 2,\ i = \{1, 2, 3, 4\}$	(Q)	24
(C)	$i\,f(i) \geq 6,\ \forall\ i\ \{1, 2, 3, 4\}$	(R)	0
(D)	$f(i) \neq (i + 1),\ \forall\ i = \{1, 2, 3, 4\}$	(S)	81
		(T)	9

10. Let a, b, c and d are four real numbers satisfying the system of equations a + b = 8, ab + c + d = 23, ad + bc = 28 and cd = 12. Now match the following

	Column - I		Column - II
(A)	a + b + c + d =	(P)	15
(B)	ab + cd =	(Q)	16
(C)	ac + bd =	(R)	27
(D)	ab =	(S)	28
		(T)	36

Integral Answer Type

11. Let E(N) denotes the sum of the even digits of n. For example E(5681) = 6 + 8 = 14.

Find $\dfrac{E(1)+E(2)+E(3)+....+E(100)}{100}$

12. Let f : I $\rightarrow$ I is defined as follows f(n) = $\begin{cases} n+3, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

suppose k is odd and f(f(f(k))) = 27 then find the sum of digits of k.

13. Find the value of $\lim\limits_{x\to\infty}\left(x+\dfrac{1}{x}\right)e^{1/x} - x$.

14. Let f : A $\rightarrow$ B be an onto function where A = {1, 2, 3, 4}, B = {x, y, z} also f(1) = x. Let M be total such functions then find the value of $\dfrac{M}{2}$.

15. Let $\sum\limits_{r=1}^{n}\int\limits_{-r(r!)\pi}^{r(r!)\pi}\dfrac{|\sin x|}{1+\pi^x}\,dx = a((n+b)! - c!)$, where a, b, c $\in$ N. Find the value of a + b + c.

16. Let P, Q, R be any three points on the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{3}=1$. Find the maximum area of ΔPQR.

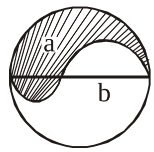
17. Let f : $R^+ \rightarrow R^+$ be a differentiable function satisfying f(xy) = $\dfrac{f(x)}{y} + \dfrac{f(y)}{x}$ $\forall$ x, y $\in R^+$ also f(1)=0,f'(1)=1. If M be the greatest value of f(x) then find the value of [M + e]. (where [.] denotes the greatest integer function).

18. Circle $(x-1)^2 + (y-1)^2 = 1$ touches X and Y axis at A and B respectively. A line y = mx intersects this circle at P and Q. If area of ΔPQB is maximum then find the value of $\left[m+\dfrac{1}{2}\right]$. (where [.] denotes the greatest integer function).

PRACTICE TEST-4

Single Choice Question :

1. The figure shown is the union of a circle and two semi-circles of diameter a and b all of whose centres are collinear. Then the ratio of the area of the shaded region to that of the unshaded region is



(A) $\frac{\pi}{ab}$ (B) $\frac{a\pi}{b}$ (C) $\frac{b}{a}$ (D) $\frac{a}{b}$

2. The value of $x \in [0, 1]$ such that tangent at (x, y) on the curve $y = x^2 + 1$ bounds maximum area with lines x = 0, y = 0, x = 1 is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{3}}$ (D) 1

3. If $I = \int \frac{x^2+20}{(x\sin x + 5\cos x)^2}\,dx$, then I equals

(A) $-\frac{x}{\cos x(x\sin x+5\cos x)} + \tan x + c$ (B) $\frac{x}{\sin x(x\sin x+5\cos x)} + \cot x + c$

(C) $(x \sin x - 5 \cos x)^{-1} \sin x + 7x + c$ (D) none of these

4. Let $g(x) = \frac{1}{\ln(1+x)} - \frac{1}{x}$, $x \in R^+$ then

(A) a < g(x) < 2 (B) $-1 < g(x) < 0$ (C) $0 < g(x) < 1$ (D) none of these

One or more than one correct :

5. If $\alpha, \beta \in \left(-\frac{\pi}{2}, 0\right)$ such that $(\sin\alpha + \sin\beta) + \frac{\sin\alpha}{\sin\beta} = 0$ and $(\sin\alpha + \sin\beta)\frac{\sin\alpha}{\sin\beta} = -1$ and $\lambda =$

$\lim\limits_{n\to\infty} \frac{1+(2\sin\alpha)^{2n}}{(2\sin\beta)^{2n}}$ then

(A) $a = -\frac{\pi}{6}$ (B) $\lambda = 2$ (C) $\alpha = -\frac{\pi}{3}$ (D) $\lambda = 1$

6. The value of $\lim\limits_{n\to\infty} \frac{\left(\sum_{r=1}^{n}\sqrt{r}\right)\left(\sum_{r=1}^{n}\frac{1}{\sqrt{r}}\right)}{\sum_{r=1}^{n} r}$ is equal to

(A) 4/3 (B) 2 (C) 8/3 (D) 5/3

7. Let the numbers a, b, c, d are in AP, then

(A) abc, a^2c, a^2c, a^2b, $\frac{a^2bc}{d}$ are in HP

(B) $a^2(bc + cd + bd)$, $b^2(ac + ad + cd)$, $c^2(ab + bd + ad)$, $d^2(ac + ab + bc)$ are in AP

(C) bcd, acd, abd, abc are in HP

(D) None of the above

8. The coordinats of the point(s) on the line x + y = 5, which is/are equidistant from the lines |x| = |y| is/are
(A) (5, 0) (B) (0, 5) (C) (–5, 0) (D) (0, –5)

Paragraph for Question nos. 9 to 11

ABC is an isosceles with AB = AC = 5 and BC = 6. Let P be a point inside the triangle ABC such that the distance from P to the base BC equals the geometric mean of the distances to the sides AB and AC.

9. The locus is the point P is
(A) a semi circle (B) a minor arc of a circle
(C) a major arc of a circle (D) a complete circle

10. The minimum distance of the point A from the locus of the point is
(A) 5/2 (B) 3/2 (C) 2 (D) None of these

11. If the tangent to the locus at B and C intersect at point P, then the area of the triangle PBC is
(A) 10 (B) 12 (C) 14 (D) 18

Paragraph for Question Nos. 12 to 14

Let P be a point in the plane of ΔABC, such that at the triangles PAB, PBC, PCA all have the same perimeter and the same area.

12. If P lies inside the ΔABC, the ΔABC
(A) must be equilateral (B) may not be equilateral
(C) must be right angled (D) None of these

13. If P lies outside the ΔABC, the ΔABC
(A) must be equilateral (B) may not be equilateral
(C) must be right angled (D) None of these

14. If P lies outside the ΔABC, then the equilateral formed by A, B, C and P is necessarily
(A) rectangle (B) square (C) rhombus (D) None of these

15. Match the following

	Column-I		Column-II
(A)	The minimum value of ab if roots of the equation $x^3 - ax^2 + bx - 2 = 0$ are positive, is	(P)	24
(B)	The number of divisors of the form $12\lambda + 6$, ($\lambda \in N$) of the number 25200 are	(Q)	3
(C)	The number of equadrilateral formed in an octagon having two adjacent sides common with the polygon are	(R)	12
(D)	The number of solution of the equation [cos x] = tan x, where [.] denotes GIF, $\forall\ x \in [0, 6\pi]$ are	(S)	18

16.

	Column-I		Column-II
(A)	Vector along a bisector of the angle between the vectors $\vec{x} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{y} = 3\hat{i} + 4\hat{j}$	(P)	$4\hat{i} + 19\hat{j} + 12\hat{k}$
(B)	Vector orthogonal to the vectors $\vec{x} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{y} = \hat{i} + 2\hat{k}$	(Q)	$2\hat{i} + 11\hat{j} - 5\hat{k}$
(C)	vector coplanar with the vectors $\vec{x} = 2\hat{i} + 3\hat{j}$ and $\vec{y} = -\hat{i} + 5\hat{j} + 6\hat{k}$	(R)	$-2\hat{i} - 5\hat{j} + \hat{k}$
(D)	vector $\vec{v}$ such that $\vec{v}$, $-3\hat{i} + 2\hat{j} + 7\hat{k}$, $\hat{i} - 2\hat{j} + 3\hat{k}$ are linearly independent	(S)	$7\hat{i} + \hat{j} + 5\hat{k}$

PRACTICE TEST-5

Single Choice Question :

1. If a, b, c are non-zero, then the system of equatin (p + a) x + py + pz = 0, px + (p + b) y + pz = 0 and px + py + (p + c) z = 0 has a non-trivials solution if

(A) $\frac{1}{p}+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ (B) $\frac{1}{p}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

(C) $p=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ (D) p + a + b + c = 1

2. If the system of equations $9x + 2\lambda y = 2\lambda$, and $x - 5y = 20$ has positive solutions for x and y, then

(A) $\lambda \in \left(-\infty,-\frac{45}{2}\right) \cup (90, \infty)$ (B) $\lambda \in (0, \infty)$

(C) $\lambda \in \left(-\frac{45}{2},90\right)$ (D) None of these

3. If $|z_1 + 2 + 2i| \le \frac{2}{3}$ and $|z_2 - 1 - 2i| \le \frac{3}{2}$, then the minimum possibnle value of $|z_2 - z_1|$ is

(A) 5 (B) $\frac{17}{6}$ (C) $\frac{49}{6}$ (D) $\frac{13}{3}$

4. The area of the region defined $1 \le |z - 1| \le 4$ and $z + \bar{z} \ge 2$ is

(A) $\frac{15\pi}{2}$ (B) 8π (C) $\frac{45\pi}{4}$ (D) $4\pi - 2$

5. Origin is an interior point of an ellipse whose foci correspond to the complex number z_1 and z_2. Then the eccentricity of ellipse is

(A) $< \frac{|z_2 - z_2|}{|z_1|+|z_2|}$ (B) $< \frac{|z_1 - z_2|}{2(|z_1|+|z_2|)}$ (C) $> \frac{|z_1 - z_2|}{|z_1|+|z_2|}$ (D) $\frac{|z_1 - z_2|}{2(|z_1|+|z_2|)}$

6. If $A(z_1)$ $B(z_2)$ $C(z_3)$ are the vertices of ΔABC in which $\angle BAC = \frac{\pi}{4}$ and $\frac{AB}{AC} = \sqrt{2}$, then $z_1 =$

(A) $z_3 + i(z_2 + z_3)$ (B) $z_3 - i(z_2 - z_3)$ (C) $z_3 + i(z_2 - z_3)$ (D) $z_3 + i(z_3 - z_1)$

One or mor than one correct :

7. Let $f(x) = (ax + b)\cos x + (cx + d)\sin x$ and $f'(x) = x \sin x$, $f(0) = 0$, $\forall\ x \in R$. Then

(A) $a = -1$ (B) $b = 0$ (C) $c = 0$ (D) $d = 1$

8. $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in I \\ x^2, & x \notin I \end{cases}$ where [.] denotes the greatest integer function. Then

(A) gof is continuous for all x (B) gof is not continuous for all x

(C) fog is continuous everywhere (D) fog is not continuous everywhere

9. In the set R of real number the function $f : R \to R$ is defined such that $|f(x) - f(y)| \le |(x - y)^3|$, $\forall$ x y$\in$ R. Then f(x) is

(A) strictly increasing (B) strictly decreasing
(C) neither increasing nor decreasing (D) a constant function

10. If A and B are events such that $P(A \cup B) \ge \frac{3}{4}$ and $\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$, then

(A) $P(A) + P(B) \le \frac{11}{8}$ (B) $P(A).P(B) \le \frac{3}{8}$ (C) $P(A) + P(B) \ge \frac{7}{8}$ (D) $P(A)\,P(B) > \frac{1}{2}$

11. Match the following

	Column-I		**Column-II**
(A)	In a ΔABC, $(a + b + c)(b + c - a) = \lambda bc$, where $\lambda \in I$, then greatest value of λ is	(P)	3
(B)	In a ΔABC, $\tan A + \tan B + \tan C = 9$. If $\tan^2 A + \tan^2 B + \tan^2 C = k$, then least value of k satisfying is	(Q)	$9(3)^{1/3}$
(C)	In a triangle ABC, then line joining the circumcentre to the incentre is parallel to BC, then value of $\cos B + \cos C$ is	(R)	1
(D)	If in a ΔABC, a = 5, b = 4 and $\cos(A - B) = \frac{31}{32}$, then the third side c is equal to	(S)	6

12. [.] represents greatest integer function is parts (A), (B) and (C)

	Column-I		**Column-II**
(A)	If $f(x) = \sin^{-1} x$ and $\lim\limits_{x \to \frac{1}{2}^+} f(3x - 4x^3) = a - 3 \lim\limits_{x \to \frac{1}{2}^+} f(x)$, then [a] =	(P)	2
(B)	If $f(x) = \tan^{-1} g(x)$ where $g(x) = \frac{3x - x^3}{1 - 3x^2}$ and $\lim\limits_{h \to 0} \frac{f(a + 3h) - f(a)}{3h} = \frac{3}{1 + a^2}$, where $\frac{-1}{\sqrt{3}} < a < \frac{1}{\sqrt{3}}$ then find $\left[\lim\limits_{h \to 0} \frac{f\left(\frac{1}{2} + 6h\right) - f\left(\frac{1}{2}\right)}{6h}\right] =$	(Q)	3
(C)	If $\cos^{-1}(4x^3 - 3x) = a + b \cos^{-1} x$ for $1 < x < \frac{-1}{2}$, then [a + b + 2] =	(R)	4
(D)	If $f(x) = \cos^{-1}(4x^3 - 3x)$ and $\lim\limits_{x \to \frac{1}{2}^+} f'(x) =$ and $\lim\limits_{x \to \frac{1}{2}^-} f'(x) = b$, then a + b + 3 =	(S)	– 2

Comprehension (Q.13 to Q.15)

Let L be the line whose equation are $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{1}$. Let P represent the point (1, 2, 3). L meets the plane p given by the equation x + y + z = 10 at A. Produce PA to B such that PA = AB. Let C be the image of B in π. Let D be a point on the plane π such that ABCD is a parallelogram. Let L_1 be the projection of L on π.

13. Coordinates of C are

(A) $\left(-\frac{1}{3},\frac{10}{3},\frac{7}{3}\right)$ (B) $\left(-\frac{1}{3},\frac{10}{3},\frac{5}{3}\right)$ (C) $\left(\frac{1}{3},\frac{10}{3},\frac{7}{3}\right)$ (D) $\left(\frac{1}{3},-\frac{10}{3},\frac{-7}{3}\right)$

14. Equation of L_1 is

(A) $\frac{7-3x}{1}=\frac{3y-10}{1}=\frac{13-3z}{1}$ (B) $\frac{7-3x}{2}=\frac{3y-10}{2}=\frac{13-3z}{1}$

(C) $\frac{7-3x}{0}=\frac{3y-10}{1}=\frac{13-3z}{2}$ (D) $\frac{7-3x}{1}=\frac{3y-10}{2}=\frac{13-3z}{2}$

15. Equation of line CD is

(A) $\frac{3x-1}{0}=\frac{3y-10}{2}=\frac{3z-7}{0}$ (B) $\frac{3x-1}{2}=\frac{3y-10}{0}\;\frac{3y-7}{0}$

(C) $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ (D) $\frac{3x-1}{1}=\frac{3y-10}{2}=\frac{3z-7}{1}$

Comprehension (Q.16 to Q.18)

It is given that $A=(\tan^{-1}x)^3+(\cot^{-1}x)^3$ where $x>0$ and $B=(\cos^{-1}t)^2+(\sin^{-1}t)^2$ where $t\in\left[0,\frac{1}{\sqrt{2}}\right]$, and

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ for $-1\le x\le 1$ and $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$ for all $x\in R$.

16. The interval in which A lies is

(A) $\left[\frac{\pi^3}{7},\frac{\pi^3}{2}\right)$ (B) $\left[\frac{\pi^3}{32},\frac{\pi^3}{8}\right)$ (C) $\left[\frac{\pi^3}{40},\frac{\pi^3}{10}\right)$ (D) None of these

17. The maximum value of B is

(A) $\frac{\pi^2}{8}$ (B) $\frac{\pi^2}{16}$ (C) $\frac{\pi^2}{4}$ (D) None of these

18. If least value of A is λ and maximum value of B is μ then $\cot^{-1}\cot\left(\frac{\lambda-\mu\pi}{\mu}\right)=$

(A) $\frac{\pi}{8}$ (B) $-\frac{\pi}{8}$ (C) $\frac{7\pi}{8}$ (D) $-\frac{7\pi}{8}$

PRACTICE TEST-6

Comprehension (Q.1 to Q.3)

Let $f(x) = \log_{\{x\}} [x]$
$g(x) = \log_{\{x\}} \{x\}$
$h(x) = \log_{\{x\}} \{x\}$
Where [.] {.} denotes the greatest integer function and fractional part function.

1. For $x \in (1, 5)$ the f(x) is not defined at how many points
(A) 5 (B) 4 (C) 3 (D) 2

2. If $A = \{x : x \in \text{domain of } f(x)\}$ and $B = \{x : x \in \text{domain of } g(x)\}$ then $\forall\ x \in (1, 5)$, A – B will be
(A) (2, 3) (B) (1, 3) (C) (1, 2) (D) None of these

3. Domain of h(x) is
(A) R (B) {I} (C) R – {I} (D) $R^+ - \{I\}$
I denotes integers

Comprehension (Q.4 to Q.6)

Let z be a complex number lying on a circle $|z| = \sqrt{2}$ a and $b = b_1 + ib_2$ (any complex number), lying on the circle then

4. Tthe equation of tangent at point 'b' is
(A) $z\bar{b} + \bar{z}b = a^2$ (B) $z\bar{b} + \bar{z}b = 2a^2$ (C) $z\bar{b} + \bar{z}b = 3a^2$ (D) $z\bar{b} + \bar{z}b = 4a^2$

5. The equation of straight line parallel to the tangent and passing through centre of circle is
(A) $z\bar{b} + \bar{z}b = 0$ (B) $2z\bar{b} + \bar{z}b = \lambda$ (C) $2z\bar{b} + 3\bar{z}b = 0$ (D) $z\bar{b} + \bar{z}b = \lambda$

6. The equation of line passing through the centre of the circle making an angle $\frac{\pi}{4}$ with the normal at 'b' are
(A) $z = \pm \frac{ib^2}{2a^2}\bar{z}$ (B) $z = \pm \frac{ib^2}{a^2}\bar{z}$ (C) $z = \pm \frac{ib^2}{3a^2}\bar{z}$ (D) $z = \pm \frac{ib^2}{4a^2}\bar{z}$

Match the Column :

7.

	Column-I		**Column-II**
(A)	$\underbrace{111....1}_{91\text{ times}}$	(P)	is a prime
(B)	1.2.3. .... n (n + 1) (n .... 3.2.1)	(Q)	is not a prime
(C)	$10^{4n} + 10^{4(n-1)} .... + 10^8 + 10^4 + 1, n \in N$	(R)	is a perfect square integer
(D)	$\underbrace{444....4}_{n\text{ times}}\ \underbrace{888....8}_{(n-1)\text{times}}9$	(S)	is a perfect square of odd integer

8.

	Column-I		**Column-II**
(A)	If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then a is	(P)	(0, 1]
(B)	Let $f(x) = (1 + b^2) x^2 + 2bx + 1$ and let m(b) the minimum value of f(x). as varies, then m(b) is	(Q)	$\left[\frac{2+\sqrt{3}}{2}, \infty\right)$
(C)	If a, b, c $\in$ R and equation $px^2 + qx + r = 0$ has two real roots α and β such that $\alpha < -1$ and $\beta > 1$, then $x^2 + \left\|\frac{q}{p}\right\| x + \frac{r}{p}$ is	(R)	< 2
(D)	The set of values of a for which bot the roots of the equation $x^2 + (2a - 1)x + a = 0$ are positive is	(S)	< 0

Integer Answer Type

9. In a class tournament where the participants were to play one game with another, two class players fell ill, having played 3 games each. If the total number of games played is 84, then the number of participants at the beginning is _____________.

10. A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all black. If P is the probability that the missed one is red. Then the value of 3P is _______.

11. If x = 1.2 $(2^2 - 1^2)$ + 2.3 $(3^2 - 2^3)$ + 3.4 $(4^2 - 3^2)$ + ... upto 50 terms, then the value of $\frac{x}{51^3}$ is ______.

12. The value of x satisfying the equation $\log_3 (\log_2 x) + \log_{1/3} (\log_{1/2} y) = 1$; $xy^2 = 9$ is ______.

13. The numbers of five digits that can be made with the digits 1, 2, 3 each of which can be used at most thrice in a number, is ______.

14. Let x, y, z be three positive real numbers such that x + y + z = 1 then the maximum value of the expression (1 – x) (2 – y) (3 – z) is ______.

15. Anubhav and Shivendra decided to meet on New Year's day eve at a place in Cannaught Place between 7 pm and 8 pm. The first one to arrive waits for 15 minutes and then leave. Assuming that each is independently equally likely to arrive at any time during the hour (between 7 pm and 8 pm). The probability that they meet, is $\frac{m}{n}$. Find (n – m).

PRACTICE TEST-7

One or more than one correct :

1. If the equation $cx^2 + bx - 2a = 0$ has no real roots and $a < \frac{b+c}{2}$ then

(A) $ac < 0$ (B) $a < 0$ (C) $\frac{c-b}{2} > a$ (D) $\frac{c+2b}{8} > a$

2. If $z = 2 + 2\sqrt{3i}$, then $z^{2n} + 2^{2n} z^n + 2^{4n}$, $n \in N$ may be equal to

(A) 2^{2n} (B) 0 (C) 3.2^{2n} (D) None of these

3. The number of ways in which we can choose 2 distinct integers from 1 to 100 such that difference between them is at most 10 is

(A) ${}^{100}C_2 - {}^{90}C_2$ (B) ${}^{100}C_{98} - {}^{90}C_{88}$ (C) ${}^{100}C_2 - {}^{90}C_{88}$ (D) None of these

4. Let n be a positive integer with $f(n) = 1! + 2! + 3! + ... + n!$ and P(x), Q(x) be polynomials in x such that $f(n + 2) = P(n) f(n + 1) + Q(n) f(n)$ for all $n \geq 1$. Then

(A) $P(x) = x + 3$ (B) $Q(x) = -x - 2$ (C) $P(x) = -x - 2$ (D) $Q(x) = x + 3$

5. If A, B, C and D are four points with positive vectors $3\hat{i}$, $3\hat{j}$, $3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ respectively, then D is the

(A) Orthocentre of ΔABC (B) Circumcentre of ΔABC
(C) Centroid of ΔABC (D) Incentre of ΔABC

6. If the first and the $(2n - 1)^{th}$ terms of an A.P., a G.P. and an H.P. are equal whereas their n^{th} terms are a, b, c respectivey, then

(A) $a = b = c$ (B) $a \geq b \geq c$ (C) $a + c = b$ (D) $ac = b^2$

7. Let f(x) is a quadratic expression with positive integral coefficients such that for every a, b $\in$ R, $\beta > \alpha$, $\int_{\alpha}^{\beta} f(x)\, dx > 0$. Let $g(t) = f''(t) f(t)$ and $g(0) = 12$, then

(A) 16 such quadratics are possible (B) f(x) = 0 has either no real root or distinct roots
(C) Minimum value of f(1) is 6 (D) Maximum value of f(1) is 11

Comprehension Type :

Comprehension (Q.8 to Q.10)

Three distinct vertices are chosen at random from the vertices of a given regular of (2n + 1) sides. Let all such choices are equally likely and the probability that the centre of the given polygon lies in the interior of the triangle determined by these three chosen random points is $\frac{5}{14}$.

8. The number of diagonals of the polygon is equal to

(A) 14 (B) 18 (C) 20 (D) 27

9. The number of points of intersection of the diagonals lying exactly inside the polygon is equal to

(A) 70 (B) 35 (C) 126 (D) 96

10. Three vertices of the polygon are chosen at random. The probability that these vertices from an isosceles triangle is

(A) 1/3 (B) 3/7 (C) 3/28 (D) None of these

Comprehension (Q.11 to Q.13)

We have to choose 11 players for cricket team from 8 batsmen, 6 bowlers, 4 allrounders and 2 wicketkeepers, in the following conditions.

11. The number of selections when at most 1 allrounder and 1 wicket keeper will play

(A) $^4C_1 \cdot {}^{14}C_{10} + {}^2C_1 \cdot {}^{14}C_{10} + {}^4C_1 \cdot {}^2C_1 \cdot {}^{14}C_9 + {}^{14}C_{11}$

(B) $^4C_1 \cdot {}^{15}C_{11} + {}^{15}C_{11}$

(C) $^4C_1 \cdot {}^{15}C_{10} + {}^{15}C_{11}$

(D) None of these

12. Number of selections when 2 particular batsman don't want to play, if a particular bowler will play

(A) $^{17}C_{10} + {}^{19}C_{11}$ (B) $^{17}C_{10} + {}^{19}C_{11} + {}^{17}C_{11}$ (C) $^{17}C_{10} + {}^{20}C_{11}$ (D) $^{19}C_{10} + {}^{19}C_{11}$

13. Number of selections when a particular batsman and a particular wicketkeeper don't want to play together

(A) $2\ {}^{18}C_{10}$ (B) $^{19}C_{11} + {}^{18}C_{10}$ (C) $^{19}C_{10} + {}^{19}C_{11}$ (D) None of these

Match the Column :

14.

Column-I		**Column-II**	
(A)	f(z) is a complex valued function f(z) = (a + ib) z where a, b $\in$ R and $\lvert a + ib\rvert = \frac{1}{\sqrt{2}}$. It has the property that f(z) is always equidistant from 0 and z, then a – b =	(P)	5
(B)	The number of all positive integers $n = 2^a 3^b$ ($a, b \ge 0$) such that n^6 does not divide 6^n is	(Q)	0
(C)	A is the region of the complex plan {z : z/4 and $4/\bar{z}$ have real and imaginary part in (0, 1)}, then [p] (where p is the area of the region A and [.] denotes the greatest integer function) is	(R)	6
(D)	If 3x + 4y + z = 5, where x, y, z $\in$ R, then minimum value of $26(x^2 + y^2 + z^2)$ is	(S)	25

15. Let $\vec{a}, \vec{b}, \vec{c}$ are the three vectors such that $|\vec{a}| = |\vec{b}| = |\vec{c}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is $\pi/3$, $\vec{b}$ and $\vec{c}$ is $\pi/3$ and $\vec{a}$ and $\vec{c}$ is $\pi/3$.

Column-I		**Column-II**	
(A)	If $\vec{a}, \vec{b}, \vec{c}$ represents adjacent edges of parallelopiped then its volume is	(P)	$\frac{2\sqrt{2}}{\sqrt{3}}$
(B)	If $\vec{a}, \vec{b}, \vec{c}$ represents adjacent edges of parallelopiped then its height is	(Q)	$\frac{2\sqrt{2}}{3}$
(C)	If $\vec{a}, \vec{b}, \vec{c}$ represents adjacent edges of tetrahedron then its volume is	(R)	$4\sqrt{2}$
(D)	If $\vec{a}, \vec{b}, \vec{c}$ represents adjacent edges of tetrahedron then its height is	(S)	$\sqrt{\frac{2}{3}}$

PRACTICE TEST-8

Comprehension Type :

Comprehension (Q.1 to Q.3)

One the ground n stones are place in a straight line. Let (r, i) denotes the distance (in mts) between the r^{th} stone and the i^{th} stone in a row. A basket is placed at a distance of 5 mts before the 1^{st} stone. A person starts from the backet and picks up a stone, comes back to basket and put the stone into abasket and then goes for the 2^{nd} stone and so on.

1. If $d(r, r + 1) = 4$, where $r \in N$, if the person had traveled a total of 650 mts. Then

(A) He has collected 10 stones in basket

(B) He has collected 11 stones in basket

(C) He has collected 12 stones in basket

(D) He has collected 13 stones in basket

2. If $d(r, r + 1) = 2r + 3$, where $r \in N$, if the person need to travel 296 mts. to pick up and i^{th} stone and put in basket then i is

(A) 10 (B) 11 (C) 12 (D) 13

3. Let $d(r, 1) = \frac{1}{(r-1)r}$, where $r \in N$ and $r \geq 2$. The person collects in total in stones in basket then

(A) i = 8, if he travelled $91\frac{7}{9}$ mts. in total

(B) i = 9, if he travelled $101\frac{4}{5}$ mts. in total

(C) i = 10, if he travelled $111\frac{9}{11}$ mts. in total

(D) i = 12, if he travelled $121\frac{5}{9}$ mts. in total

Comprehension (Q.4 to Q.6)

Tangents drawn to the parabola $y^2 = 8x$ at the points $P(t_1)$ and $Q(t_2)$ intersects at a point T and normals at P and Q intersects at a point R such that t_1 and t_2 are the roots of equation $t^2 + at + 2 = 0$; $|a| > 2\sqrt{2}$.

4. Locus of R is

(A) $y^2 = 4x$ (B) $y^2 = -4x$ (C) $2y^2 = -x$ (D) $x^2 = 8$

5. Locus of circumcentre of ΔPTQ is

(A) $y^2 = x - 2$ (B) $2y^2 = x + 2$ (C) $2y^2 = x - 2$ (D) $2y^2 = 3(x - 2)$

6. The point on the curve given by locus in part(2) which is at a minimum distinct from the line $y = x + 1$

(A) (6, 2) (B) $\left(\frac{9}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{13}{2}, \frac{3}{2}\right)$ (D) (2, 0)

Match the Column :

7. **Column-I** | **Column-II**

(A) The triangle PQR is inscribed in the circie $x^2 + y^2 = 169$. If Q and R have coordinates (5, 12) and (–12, 5) respectively find $\angle QPR$ — (P) $\pi/4$

(B) What is the angle between the line joining origin to the point of intersection of the line $4x + 3y = 24$ with circle $(x - 3)^2 + (y - 4)^2 = 25$ — (Q) $\pi/2$

(C) Two parallel tangents drawn to given circle are cut by a third tangent. The angle subtended by the third tangent at the centre is — (R) π

(D) For a circle if a chord is drawn along the point of contact of tangents drawn from a point P. If the chord subtends an angle $\pi/2$ then find the angle at P. — (S) $\pi/6$

8. **Column-I** | **Column-II**

(A) Point (4, 1) is reflected about $y = x$ and then transformed through distance 2 units along positive direction of x-axis. it also rotates through an angle θ about the origin in counter clockwise direction. If its final position is $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ then $\tan\theta$ is. — (P) 3

(B) Tangent to ellipse $\frac{8^2}{8} + \frac{y^2}{324} = 1$ at $(2\sqrt{3}\cos\theta, 18\sin\theta)$ $\theta \in (0, \pi/2)$. The value of $\tan^2\theta$ such that sum of intercepts on axes made by this tangent is minimum is — (Q) 9

(C) If the line $2x + \sqrt{6}\,y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$. Then slope of line joining origin and point of contact is — (R) 1

(D) Tangent at P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets a straight line at point Q on y-axis where length PQ = 5. Then, length OQ = ?, (where O is the origin) — (S) $-\frac{\sqrt{3}}{2\sqrt{2}}$

Integer Answer Type :

9. Let $P(\alpha_1, \beta_1)$, $Q(\alpha_2, \beta_2)$ and $R(\alpha_3, \beta_3)$ be the centroid, orthocentre and circumcentre of a scelene triangle having its vertices on the curve $y^2 = x^3$, then $\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3}$ is equal to ______.

10. If the equation of the curve on reflection of the ellipse $\frac{(x-4)^2}{16} + \frac{(x-3)^2}{9} = 1$ about the line $x - y - 2 = 0$ is $16x^2 + 9y^2 + k_1 x - 36y + k_2 = 0$, then $k_1 + k_2$ is ______.

11. The base AB of a triangle is 1 and height h of C from AB is less than or equal to 1/2. The maximum value of the 4 times the product of the altitudes of the triangle is ______.

12. In a triangle ABC, $c \geq a$, $c \geq b$ and $a^2 + b^2 = 2Rc$, (Notations have their usual meaning). The value of the angle C is π/k, then k is

PRACTICE TEST-9

Single Choice Quesion :

1. Equation of the image of the line $x + y = \sin^{-1}(a^6 + 1) + \cos^{-1}(a^4 + 1) - \tan^{-1}(a^2 + 1)$, $a \in R$ about x-axis is given by

(A) $x - y = 0$ (B) $x - y = \frac{\pi}{2}$ (C) $x - y = \pi$ (D) $x - y = \frac{\pi}{4}$

2. If $f(\theta) = \frac{1 - \sin 2\theta + \cos 2\theta}{2\cos 2\theta}$ then value of $f(11^\circ) \cdot f(34^\circ)$ equals

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) 1

3. $\int_0^{\pi/2} \frac{\cos\theta - \sin\theta}{(1+\cos\theta)(1+\sin\theta)} d\theta$ equals

(A) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (B) $\cos^{-1}(0)$ (C) $\cos^{-1}(1)$ (D) $\cos^{-1}(-1)$

4. The roots of the equation $x^3 - 10x + 11 = 0$ are u, v, and w. The value of $(\tan^{-1} u + \tan^{-1} v + \tan^{-1} w)$ equals

(A) – 1 (B) $\tan^{-1}(1)$ (C) 1 (D) $\tan^{-1}(-1)$

5. If the equation $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))} = 0$ has a solution for ‘x’ when $c < y < b$, $y \neq a$, where ‘b’ is as large as possible and ‘c’ is as small as possible, then the value of $(a + b + c)$ is equal to

(A) 18 (B) 19 (C) 20 (D) 21

6. If the equation $2x^2 + 4xy + 7y^2 - 12x - 2y + t = 0$ where ‘t’ is a parameter has exactly one real solution of the form (x, y). Then the sum of (x + y) is equal to

(A) 3 (B) 5 (C) – 5 (D) – 3

Comprehension (Q.7 to Q.9)

Consider a rational function $f(x) = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$ and a quadratic function $g(x) = (1 + m)x^2 - 2(1 + 3m)x - 2(1 + m)$ where m is a parameter.

7. Which one of the following statement does not hold good for the rational function f(x) ?

(A) It is a continuous function

(B) It has only one asymptote

(C) It has exactly one maxima and one minima

(D) f(x) is monotonic in $(0, \infty)$

8. If $\cot(\cot^{-1} f(x)) = k$, has exactly two distinct real solutions then the integral value of ‘k’ can be

(A) 0 (B) – 1 (C) 1 (D) 5

9. If the range of the function f(x) lies between the roots of g(x) = 0 fthenthe number of integral values of m equals

(A) 7 (B) 8 (C) 9 (D) 10

One or more than one correct :

10. For the function $f(x) = (x^2 + bx + c)\, e^x$ and $g(x) = (x^2 + bx + c)\, e^x (2x + b)$. Which of the following holds good ?

(A) if $f(x) > 0$ for all real $x \not\Rightarrow g(x) > 0$ (B) if $f(x) > 0$ for all real □$\Rightarrow g(x) > 0$

(C) if $g(x) > 0$ for all real $x \Rightarrow f(x) > 0$ (D) if $g(x) > 0$ for all real $x \not\Rightarrow f(x) > 0$

11. For any odd integer $n \geq 1$, if the value of the sum

$n^3 - (n-1)^3 + (n-2)^3 + ... + (-1)^{n-1}(1^3)$ equals 208 then 'n' can not be

(A) 5 (B) 7 (C) 9 (D) 11

12. Which of the following equations have no real real solutions ?

(A) $x^2 - 2x + 5 + \pi^x = 0$ (B) $\log_{1.5}(\cos^{-1} x - \mathrm{sgn}(e^x)) = 2$

(C) $x^4 - 2x^2 \sin^2 \dfrac{\pi x}{2} + 1 = 0$ (D) $\tan\left(x + \dfrac{\pi}{6}\right) = 2 \tan x$

Match the Column :

13. **Column-I** **Column-II**

(A) $f : R \to R$ is defined as $f(x) = \begin{cases} x^2 + kx + 3 & \text{for } x \leq 0 \\ 2kx + 3 & \text{for } x < 0 \end{cases}$ (P) 2

If f(x) is injective then 'k' can be equal to (Q) 5

(B) If $\lim\limits_{x \to 2} \dfrac{f(x) - 9}{x - 2} = 3$ then $\lim\limits_{x \to 2} f(x)$, is (R) 9

(C) If $\lim\limits_{x \to \infty} \dfrac{7^{kx} + 8}{7^{5x} + 6}$ does not exist then 'k' can be (S) 12

13. **Column-I** **Column-II**

(A) If both roots of f(x) = 0 are confined in (–1, 1) then (P) $\left(\dfrac{2}{5}, \infty\right)$

(B) Exactly one root of f(x) = 0 lies in (–1, 1) (Q) ϕ

(C) Both roots of f(x) = 0 are greater than 1 (R) $\left(-\dfrac{1}{17}, \dfrac{1}{3}\right)$

(D) One root of f(x) = 0 is greater than 1 and other root is (S) $\left(-\infty, -\dfrac{1}{17}\right] \cup \left[\dfrac{1}{3}, \infty\right)$

f(x) = 0 is less than – 1 (R) $\left(-\infty, -\dfrac{1}{17}\right] \cup \left(\dfrac{1}{3}, \infty\right)$

Subjective :

15. Polynomial P(x) contains only terms of odd degree. When P(x) is divided by $(x - 3)$, the remainder is 6. If P(x) is divided by $(x^2 - 9)$ then remainder is 9(x). Find the value of g(2).

16. If the sum of the roots of the equation $\cos 4x + 6 = 7 \cos 2x$ in the interval [0, 314] is $k\pi$, $k \in R$. Find k.

PRACTICE TEST-10

Single Choice Quesion :

1. Let $P(x) = a_1 x + a_2 x^2 + a_3 x^3 + ... + a_{100} x^{100}$, where $a_1 = 1$ and $a_i \in R \ \forall \ i = 2, 3, 4, ... , 100$.

Then $\lim\limits_{x\to 0} \dfrac{\sqrt[100]{1+P(x)}-1}{x}$ has the value equal to

(A) 100 (B) $\dfrac{1}{100}$ (C) 1 (D) 5050

2. $\displaystyle\int_0^{\pi/4} \frac{e^{\sec^2 x} \sin x}{\cos^3 x} dx$ equals

(A) $\dfrac{e^2}{2}$ (B) $\dfrac{e^2-1}{2}$ (C) $\dfrac{e^2+1}{2}$ (D) $\dfrac{e^2-e}{2}$

3. Let A(–1, 0), B(0, 1, 0), C(0, 0, –1) are the vetives of ΔABC. If R and r denote the circumradius and inradius of ΔABC, then the value of $(R^2 + r^2)$ is equal to

(A) 5/6 (B) 1/6 (C) 2/3 (D) 1/3

4. Most general value of 'c' for which the equation $\displaystyle\int_0^{c/2} (\cos x - \cos(x-c))\ dx = \int_{\pi}^{(\pi/2)+c} \cos(x-c)\ dx$ holds good, are given by

(A) $n\pi + (-1)^n \pi/3$ (B) $n\pi + (-1)^n \pi/6$ (C) $2n\pi + (-1)^n \pi/3$ (D) $2n\pi + (-1)^n \pi/6$

5. If sin (sin x + cos x) = cos (cos x – sin x), then the greatest possible value of sin x, is

(A) $\dfrac{1}{\sqrt{2}}$ (B) 1 (C) $\dfrac{\sqrt{16-\pi^2}}{4}$ (D) $\dfrac{\pi}{4}$

6. Minimum distance between the curve $f(x) = e^x$ and $g(x) = \ell n\ x$ is

(A) $\sqrt{2}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) 1 (D) $2(\sqrt{2}-1)$

7. Number of terms common to the two sequences 17, 21, 25, ... , 417 and 16, 21, 26, ... , 466 isd

(A) 19 (B) 20 (C) 21 (D) 22

8. Two loaded dice each have the property that 2 or 4 is three times as likely to appear as 1, 3, 5 or 6 on each roll. /when two such dice are rolled, the probability of obtaininga total of 7, is

(A) 1/8 (B) 1/7 (C) 7/50 (D) 7/25

9. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ, then the maximum value of θ is

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{2}$ (C) $\dfrac{2\pi}{3}$ (D) $\dfrac{5\pi}{6}$

10. If $x_1, x_2, x_3, ... x_{n-1}$ be a zero's of the polynomial $P(x) = x^n + \alpha x + \beta$,
where $x_i \neq x_j \ \forall \ i \ \& \ j = 1, 2, 3, ... (n-1)$.
The value of $Q(x) = (x_1 - x_2)(x_1 - x_3)(x_1 - x_4) ...... (x_1 - x_{n-1})$, is

(A) $n(n-1)x_1^{n-2}$ (B) ${}^nC_2\, x_1^{n-2}$ (C) $(nx_1^{n-1}) + \alpha$ (D) zero

11. A variable line $x \cos \alpha + y \sin \alpha = p$ where p is a constant, mets the x and y axis at A and B respectively. The locus of a point R whch divide the line segment AB extrenally in the ratio 3 : 2 is given by
(A) $9x^{-2} - 4y^{-2} = p^{-2}$ (B) $4x^{-2} - 9y^{-2} = p^{-2}$ (C) $9x^{-2} + 4y^{-2} = p^{-2}$ (D) $4x^{-2} + 9y^{-2} = p^{-2}$

12. For $x \geq 0$, the smalest value of the function $f(x) = \dfrac{4x^2 + 8x + 13}{6(1+x)}$, is

(A) 1 (B) 2 (C) $\dfrac{25}{12}$ (D) $\dfrac{13}{6}$

13. How many numbers from 1 ro 1000 are divisible by 60 but not byu 24 ?
(A) 8 (B) 12 (C) 16 (D) 24

14. $\lim\limits_{n\to\infty} \left(1-\dfrac{1}{2^2}\right)\left(1-\dfrac{1}{3^2}\right)\left(1-\dfrac{1}{4^2}\right) .... \left(1-\dfrac{1}{n^2}\right)$ equals
(A) 3/8 (B) 1/2 (C) 1/4 (D) 1/8

15. In ΔABC, angle A is 120º, BC + CA = 20 and AB + BC = 21, then the length of the side BC, equals
(A) 13 (B) 15 (C) 17 (D) 19

16. A line passes through the point $A(\hat{i} + 2\hat{j} + 3\hat{k})$ and is parallel to the vector $\vec{V} = (\hat{i} + \hat{j} + \hat{k})$. The shortest distance from the origin, of the line is
(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$

17. Equation of a straight line which passes through the point of intersection of the lines $3x - 4y + 6 = 0$ and $x + y + 2$ and has equal intercepts on the coordinates axes, is
(A) $x - y + 2 = 0$ (B) $2x + 2y + 3 = 0$ (C) $x + y + 2 = 0$ (D) no such line can be found out

18. If f be a continuous function on [0, 1], differentiable in (0, 1) such that f(1) = 0, then their exists some $c \in (0, 1)$ such that
(A) $c f'(c) - f(c) = 0$ (B) $f'(c) + cf(c) = 0$ (C) $f'(c) - cf(c) = 0$ (D) $cf'(c) + f(c) = 0$

19. If $f(x) = (2x - 3\pi)^5 + \dfrac{4}{3} x + \cos x$ and g is the inverse function of f, then $g'(2\pi)$ is equal to

(A) $\dfrac{7}{3}$ (B) $\dfrac{3}{7}$ (C) $\dfrac{30\pi^4 + 4}{3}$ (D) $\dfrac{3}{30\pi^4 + 4}$

Comprehension (Q.20 to Q.22)

Consider a polynomial y = P(x) of the least degree passing through A(–1, 1) and whose graph has two points of inflextion B(1, 2) and C with abscissa 0 at which the curve is inclined to the positive axis of abscissas at an angle of $\sec^{-1} \sqrt{2}$.

20. If $\int_{-2}^{2} P(x)\, dx = \dfrac{k}{5}$, $k \in N$ then k equals
(A) 17 (B) 24 (C) 32 (D) 41

21. The value of P(–1) equals
(A) – 1 (B) 0 (C) 1 (D) 2

22. The area of ΔABC equals
(A) 1/2 (B) 1/4 (C) 1/8 (D) 1/12

One or more than one correct :

23. The function $f(x) = \frac{\tan x}{\tan x}$ is

(A) both many one and even function (B) $\lim_{x\to 0}$ f(x) equals

(C) periodic function (D) $\lim_{x\to \pi/2}$ f(x) does not exists

24. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then possible integer(s) in the range of $\frac{3|\vec{a}+\vec{b}|}{2} + 2|\vec{a} - \vec{b}|$ is

(A) 2 (B) 3 (C) 4 (D) 5

25. Consider a real valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + tf(t))\, dt$. If M and m are maximum and minimum value of the function f, then

(A) $\frac{M}{m} = 3$ (B) $M - m = 2\pi + 1$ (C) $M + m = 4(\pi + 1)$ (D) $Mm = 2(\pi^2 + 1)$

26. If $0 \le \theta \le \pi$ and $\sin \frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$, then possible values of $\tan \theta$, is

(A) 4/3 (B) 0 (C) – 3/4 (D) – 4/3

27. Let $I = \int_{k\pi}^{(k=1)\pi} \frac{|\sin 2x|\, dx}{|\sin x| + |\cos x|}$, $(k \in N)$ and $J = \int_0^{\pi/4} \frac{dx}{\sin x + \cos x'}$ then which of the following hold(s) good?

(A) $I = \int_0^{\pi/2} \frac{\sin 2x\, dx}{\sin x + \cos x}$ (B) I = 4 – 4J (C) I = 4 – 2J (D) I = 2 – 2J

28. Which of the following hold(s) good for the function $f(x) = 2x - 3x^{2/3}$?

I The function has an extremum point at x = 0

II The function has a critical numberat x = 1

III The graph is concave down at every point in $(-\infty, \infty)$

(A) Statement I (B) Statement II (C) Statement III (D) I, II and III are true

29. If p, q, r are positive rational numbers such that p > q > r and the quadratic equation $(p + q - 2r) x^2 + (q + r - 2p) x + (r + p - 2q) = 0$ has a root in (–1, 0) then which of the following statement hold(s) good ?

(A) Equation $px^2 + 2qx + r = 0$

(B) Both roots of the given quadratic equation are rational

(C) Equation $px^2 + 2qx + r = 0$ has real and distinct roots

(D) $\frac{r+p}{q} < 2$

30. Number of real values of x satisfying the equation $3^{x+2} + 3^{2-x} = 82$ is not equal to

(A) Number of real solutions of the equation $(|x| + 1)^2 = 4|x| + 9$, such that the quantity $\ell n\,(5 - 2x)$ is a real number

(B) The value of the expression $\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$ when simplified

(C) Number of real solutions of the equation, $2x\, \ell n\, x + x - 1 = 0$

(D) The value of 'm' if a line of gradient m passes through the points (m, – 9) and (7, m)

PRACTICE TEST-11

Single Choice Quesion :

Let f(x) be a differentiable function and satisfy $f(0) = 2$, $f'(0) = 3$ and $f''(x) = f(x)$

1. The range of the function f(x) is

(A) $(0, \infty)$ (B) $(-\infty, \infty)$ (C) $[1, \infty)$ (D) $(-\infty, -1]$

2. The value of the function when $x = \ell n\, 2$ is

(A) $\frac{19}{4}$ (B) $\frac{9}{4}$ (C) $\frac{19}{2}$ (D) 6

3. The area enclosed by y = f(x) in the 2^{nd} quadrant is

(A) $3 + \frac{1}{2}\, \ell n \sqrt{5}$ (B) $2 + \frac{1}{2}\, \ell n 5$ (C) $3 - \sqrt{5}$ (D) 3

Assertion & Reason :

4. **Statement-1 :** If f(x) is differentiable in [0, 1] such that f(0) = f(1) = 0, then for any $\lambda \in R$, there exists c such that $f'(c) = \lambda f(c)$, $0 < c < 1$.

Statement-2 : If g(x) is differentiable in [0, 1] where g(0) = g(1), then there exists c such that g'(c) = 0,

$0 < c < 1$.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

5. **Statement-1 :** If f(x) is differentiable in [0, 1] such that f(0) = f(1) = 0, then for any $\lambda \in R$, there exists c such that $f'(c) = \lambda f(c)$, $0 < c < 1$.

Statement-2 : If g(x) is differentiable in [0, 1] where g(0) = g(1), then there exists c such that g'(c) = 0,

$0 < c < 1$.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

6. **Statement-1 :** Let u, v, w satisfy the equations uvw = – 6, uv + vw = – 5, u + v + w = 2 where u > v > w, then the set of value(s) of 'a' for which the points P(u, –w) and Q(v, a^2) lies onthe same side ofthe line $4x - y + 5 = 0$ are given by (–3, 3).

Statement-2 : If two points $M(x_1, y_1)$ and $N(x_2, y_2)$ lies on the same side of the line ax + by + c = 0, then

$(ax_1 + by_1 + c)(ax_2 + by_2 + c) > 0$.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

7. Consider a curve C : $y = \cos^{-1}(2x - 1)$ and a straight line $L = 2px - 4y + 2\pi - p = 0$.

Statement-1 : The set of values of 'p' for which the line L intersects the curve at three distinct points is $[-2\pi, -4]$

Statement-2 : The line L is always passing through point of inflection of the curve C.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

Mathc the Column :

8.

	Column-I		**Column-II**
(A)	Possible integral value(s) of 'm' for which the equation $z^3 + (3 + i) z^2 - 3z - (m + i) = 0$ has at least one real root is	(P)	0
(B)	The value of the definite integral $\int_{-3\pi/4}^{5\pi/4} \frac{\cos x + \sin x}{1 + e^{x-(\pi/4)}} dx$ equals	(Q)	1
(C)	Let $a_1, a_2, a_3, ..., A_{201}$ ARE IN G.P. with $a_{101} = 25$ and $\sum_{i=1}^{201} a_i = 625$. Then the value of $\sum_{i=1}^{201} \frac{1}{a_i}$ equals	(R)	3
(D)	If $\cos\theta_1 = \frac{2\cos\theta_2 - 1}{2 - \cos\theta_2}$ where $\theta_1, \theta_2 \in (0, \pi)$	(S)	4
	then the value of $\frac{\tan^2 \frac{\theta_1}{\theta_1}}{\tan^2 \frac{\theta_2}{2}}$ equals	(T)	5

9.

	Column-I		**Column-II**
(A)	Let $f(x)(\sin g(x) - \cos g(x)) + \ell$, where λ is constant of integration is the primitive of $\sin(\ell nx)$. If $a = \lim_{x\to 2} f(x)$ and $b = g(e^5) + g(e^3) - 6$, then point (a, b) lies on the curve	(P)	$y = 2x$
(B)	If the equation $\sin^{-1}(x^2 + x + 1) + \cos^{-1}(\ell x + 1) = \frac{\pi}{2}$ has exactly two solution for $\lambda \in [a, b)$, then point (a, b) lies on the curve	(Q)	$y = x + 1$
(C)	Let $f(n) = \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$. If $a = f(1)$ and $2b = f(2)$, then point (a, b) lies on the curve	(R)	$y^2 - x^2 = 3$
(D)	If a and b are real numbers and $1 + \sqrt{-1}$ is a root of the equation $x^3 - 3x^2 - bx + a = 0$, then point (a, b) lies on the curve	(S)	$xy = 0$
		(T)	$y^2 = 4x$

Subjective :

10. Given $f^2(x) + g^2(x) + h^2(x) \le 9$ and $U(x) = 3f(x) + 4g(x) + 10h(x)$. where f(x), g(x) and h(x) are continuous $\forall\, x \in R$. If maximum vaue of U(x) is $\sqrt{N}$, then find N.

11. Let P(x) be a polynomial of degree 5 having extremum at x = – 1. 1 and $\lim\limits_{x\to 0}\left(\frac{P(x)}{x^3} - 2\right) = 4$. If M and m are the maximum and minimum value of the funcxtion y = P'(x) on the set A = {x{$x^2 + 6 \le 5x$} then find $\frac{m}{M}$.

12. Let f(x) be a non-constant thrice differentiable function defined on $(-\infty, \infty)$ such that f(x) = f(6 – x) and f'(0) = 0 = f'(2) = f'(5). Determine the minimum number of zeroes of $g(x) = (f''(x))^2 + (f'(x)\, f'''(x))$ in the interval [0, 6].

13. Let $f(x) = x^3 - \frac{3x^2}{2} + x + \frac{1}{4}$. Find the value of $\left(\int_{1/4}^{3/4} f(f(x))\,dx\right)^{-1}$.

14. In ΔABC, a point P is chosen on side $\vec{AB}$ so that AP : PB = 1 : 4 and a point Q is chosen on the side $\vec{BC}$ so that CQ + QB = 1 : 3. Segment $\vec{CP}$ and $\vec{AQ}$ intersect at M. If the ratio $\frac{MC}{PC}$ is expressed as a rational number of the lowest terms as $\frac{a}{b}$, then find (a + b).

15. Let two parallel lines L_1 and L_2 with positive slope are tangent to the circle $C_1 : x^2 + y^2 - 2x - 16y + 64 = 0$. If L_1 is also tangent to the circle $C_2 : x^2 + y^2 - 2x + 2y - 2 = 0$ and equation of L_2 is $a\sqrt{a}\, x - by + c - a\sqrt{a} = 0$ where a, b, c $\in$ N, then find the value of (a + b + c).

PRACTICE TEST-12

Single Choice Quesion :

1. The range of k for which the inequality $k \cos^2 x - k \cos x + 1 \geq 0\ \forall\ x \in (-\infty, \infty)$, is

(A) $k > -\frac{1}{2}$ (B) $k > 4$ (C) $-\frac{1}{2} \leq k \leq 4$ (D) $\frac{1}{2} \leq k \leq 5$

2. If [x] and {x} denotes the greatest integer function less than or equal to x and fractional part function respectivey, then the number of real x, satisfying the equation $(x - 2)[x] = \{x\} - 1$, is

(A) 0 (B) 1 (C) 2 (D) infinite

3. Let $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$ and g(x) be the inverse of f(x), then which one of the following holds good ?

(A) $2g'' = g^2$ (B) $2G'' = 3g^2$ (C) $3g'' = 2g^2$ (D) $3g'' = g^2$

4. If $g(x) = \int_1^x e^{t^2}\, dt$ then the value of $\int_3^{x^3} e^{t^2}\, dt$ equals

(A) $g(x^3) - g(3)$ (B) $g(x^3) + g(3)$ (C) $g(x^3) - 3$ (D) $g(x^3) - 3g(x)$

5. Let $g(x) = ax + b$, where $a < 0$ and g is defined from [1, 3] onto [0, 2] then the value of $\cos(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|))$ is equal to

(A) g(1) (B) g(2) (C) g(3) (D) g(1) + g(3)

6. Let A be the set of all 3 × 3 skew symmetric matrices whose entries are either – 1, 0 or 1. If there are exatcly three 0's three 1's and three (–1)'s, the the number of such matrices, is

(A) 3 (B) 6 (C) 8 (D) 9

Comprehension (Q.7 to Q.10)

Consider two curves y = f(x) passing through (0, 1) and the curve $g(x) = \int_{-\infty}^{x} f(t)\, dt$ passing through (0, 1/2). The tangent drawn to both curves at the points with equal abscissas intersect on the x-axis.

7. The value of f'(0) equals

(A) 0 (B) 1/2 (C) 2 (D) 1

8. The value of $g'\left(\frac{1}{2}\right)$ equals

(A) e (B) e/2 (C) 2e (D) 1

9. $\lim\limits_{x \to 0} \frac{f^2(x) - 1}{x}$ equals

(A) 1 (B) 2 (C) 3 (D) 4

10. The area bounded by the x-axis, the tangent and normal to the curve y = f(x) at the point where it cuts the y-axis, is

(A) 3/4 (B) 1 (C) 5/4 3/2

One or more than one correct :

11. If the expression $(1 + ir)^3$ is of the form of s(1 + i) for some real 's' where 'r' is also real and $i = \sqrt{-1}$, then the value of 'r' can be

(A) $\cot \frac{\pi}{8}$ (B) $\sec \pi$ (C) $\tan \frac{\pi}{12}$ (D) $\tan \frac{5\pi}{12}$

12. Consider a real valued continuous function f(x) defined on the interval [a, b]. Which of the following statements does not hold(s) good ?

(A) If $f(x) \geq 0$ on [a, b] then $\int_a^b f(x)\, dx \leq \int_a^b f^2(x)\, dx$

(B) If f(x) is increasing on [a, b]l, then $f^2(x)$ is increasing on [a, b]

(C) If f(x) is increasing on [a, b]l, then $(x) \geq 0$ on (a, b)

(D) If f(x) attains a minimum at x = c where a < c < b, then f'(c) = 0

13. If a, b, c $\in R^+$ then $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{n}{(k+an)(k+bn)}$ is equal to

(A) $\frac{1}{a-b} \ln \frac{b(b+1)}{a(a+1)}$ if $a \neq b$ (B) $\frac{1}{a-b} \ln \frac{a(b+1)}{b(a+1)}$ if $a \neq b$

(C) non existent if a = b (D) $\frac{1}{a(1+a)}$ if a = b

Match the Column :

14. **Column-I** **Column-II**

	Column-I		Column-II
(A)	The 5^{th} and 8^{th} terms of a geometric sequence of real numbers are 7 ! and 8 ! respectively. If the sum of first n terms of the G.P. is 2205, then n equals	(P)	1
(B)	If $x \in (0, 2\pi)$ and $y \in (0, 2\pi)$, then the number of distinct ordered pairs (x, y) satisfying the equation $9 \cos^2 x + \sec^2 y - 6 \cos x - 4 \sec y + 5 = 0$, is	(Q)	2
(C)	Let $f(x) = ax^2 + bx + c$ (where $a \neq 0$ and a, b, c $\in$ R) and $(a + c)^2 < b^2$, then the number of distinct values of x in (–1, 1) satisfying the equation f(x) = 0 will be	(R)	3
		(S)	4

15. In a ΔABC, BC = 2, CA = 1 + $\sqrt{3}$ and $\angle C = 60°$. Feet of the perpendicular from A, B and C on the opposite sides BC, CA and AB are D, E and F respectively and are concurrent at P. Now match the entries of **column-I** with respective entries of **column-II**.

	Column-I		Column-II
(A)	Radius of the circle circumscribing the ΔDEF, is	(P)	$(\sqrt{6} - \sqrt{2})/4$
(B)	Area of the ΔDEF, is	(Q)	$\frac{1}{\sqrt{2}}$
(C)	Radius of the circle inscribed in the ΔDEF, is	(R)	$\frac{\sqrt{3}}{4}$
		(S)	$(\sqrt{6} + \sqrt{2})/4$

Subjective :

16. If $f(x) = x + \int_0^1 t(x+t)\, f(t)\, dt$, then the value of the definite integral $\int_0^1 f(x)\, dx$ can be expressed in the form of rational as p/q (where p and q are coprime). Find (p + q)

17. There is a point (p, q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = -8/x$, where p > 0 and r > 0. If the line through (p, q) and (r, s) is also tangent to both the curves at these pointys respectively, then find the vlaue of (p + r)

PRACTICE TEST-13

Single Choice Quesion :

1. If r, s and non zero roots of $a_0 + a_1x + a_2x^2 = 0$ ($a_0, a_1, a_2 \in R$ and $a_2 \neq 0$), then the equality $a_0 + a_1x + a_2x^2 = a_0 \left(1-\frac{x}{r}\right)\left(1-\frac{x}{s}\right)$ holds

(A) for all values of x, $a_0 \neq 0$
(B) only when x = 0
(C) only when x = r or x = s
(D) only when x = r or x = s, $a_0 \neq 0$

2. If $y = \frac{x}{\sqrt{a^2-1}} - \frac{2}{\sqrt{a^2-1}} \tan^{-1}\left(\frac{\sin x}{a+\sqrt{a^2-1}+\cos x}\right)$ where $a \in (-\infty, -1) \cup (1, \infty)$ then $y'\left(\frac{\pi}{2}\right)$ equals

(A) $\frac{1}{a}$
(B) $\frac{2}{a}$
(C) $\frac{1}{2a}$
(D) a

3. For $\theta \in \left(0, \frac{\pi}{2}\right)$, the value of definite integral $\int_0^{\theta} \ln(1+\tan\theta \tan x)\, dx$ is equal to

(A) θ on (sec θ)
(B) θ on (cosec θ)
(C) $\frac{\theta \ln 2}{2}$
(D) 2θ ln sec θ

Comprehension (Q.4 to Q.6)

Consider $f(x) = 4x^4 - 24x^3 + 31x^2 + 6x - 8$ be a polynomial function and α, β, γ, δ are the roots of the equation f(x) = 0, where α < β < γ < δ. Let sum of two roots of the equation f(x) = 0 vanishes.

4. The value of the expression $\delta^\beta + \frac{1}{\delta^\alpha} + \delta^\gamma + \gamma^\delta$ is

(A) 36
(B) 35
(C) 20
(D) 16

5. $\int \left(\frac{x-\delta}{x-\gamma}\right)^{\alpha+\beta+\delta} dx$ is

(A) $x - 8\ell n\,|x-2| - \frac{24}{x-2} + \frac{16}{(x-2)^2} - \frac{16}{3(x-2)^3} + C$

(B) $x - 16\ell n\,|x-2| - \frac{24}{(x-2)} + \frac{32}{(x-2)^2} - \frac{16}{(x-2)^3} + C$

(C) $x - 16\ell n\,|x-2| - \frac{24}{(x-2)} + \frac{16}{(x-2)^2} - \frac{16}{3(x-2)^3} + C$

(D) $x - 8\ell n\,|x-2| - \frac{24}{(x-2)} + \frac{16}{(x-2)^2} - \frac{16}{3(x-2)^3} + C$

6. $\int_{2\alpha}^{2\beta} \frac{x^{\delta+1} - 5x^{\gamma+1} + 2\beta|x| + 1}{x^2 + 4\beta|x| + 1}\, dx$ is

(A) $\ell n2$
(B) $2\ell n2$
(C) $\frac{1}{2}\ \ell n2$
(D) $\ell n\,\frac{1}{2}$

Comprehension (Q.7 to Q.9)

a, b, c are the sides of ΔABC satisfying $\log\left(1+\frac{c}{a}\right) + \log a - \log b = \log 2$.

Also the quadratic equation $a(1 - x^2) + 2bx + c(1 + x^2) = 0$ has two equal roots.

7. a, b, c are in

(A) A.P. (B) G.P. (C) H.P. (D) None

8. Measure of angle C is

(A) 30º (B) 45º (C) 60º (D) 90º

9. The value of (sin A + sin B + sin C) is equal to

(A) 5/2 (B) 12/5 (C) 8/3 (D) 2

Assertion & Reason :

10. **Statement-1 :** Only one straight line can be drawn passing through the origin, at equal distances from the points A(2, 2) and B(4, 0)

Statement-2 : Only one straight line can be drawn passing through two given points.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

11. **Statement-1 :** If the circles $x^2 + y^2 - 2x + 4y + 4 = 0$ and $x^2 + y^2 + 4x - 2y + c = 0$ intersect such that the common chod is longest, then c = – 14.

Statement-2 : If two circles intersect, the common chord is longest if it is a diameter of the smaller circle.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

12. Consider two functions f(x) = sin x and g(x) = |f(x)|
Statement-1 : The function h(x) = f(x) is not differentiable in $[0, 2\pi]$

Statement-2 : f(x) is differentiable and g(x) is not differentiable in $[0, 2\pi]$

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

One or more than one correct :

13. Let $f(x) = \text{sgn}(\text{are cot } x) + \tan\left(\frac{\pi}{2}[x]\right)$, where [x] is the greatest integer function less than or equal to x.

Then which of the following alternatives is/are true ?

(A) f(x) is many one but not even function (B) f(x) is periodic function
(C) f(x) is bounded function (D) Graph of f(x) remains above x-axis

14. Let $I_n = \int_0^{\sqrt{3}} \frac{dx}{1+x^n}$ (n = 1, 2, 3, ...) and $\lim_{n\to\infty} I_n = I_0$ (say), then which of the following statement(s) is/are correct ? (Given : e = 2.71828)

(A) $I_1 > I_0$ (B) $I_2 < I_0$ (C) $I_0 + I_1 + I_2 > 3$ (D) $I_0 + I_1 > 2$

15. The possible radius of a circle whose centre is at the origin and which touches the circle $x^2 + y^2 - 6x - 8y + 21 = 0$.

(A) 2 (B) 3 (C) 5 (D) 7

Match the Column :

16.

	Column-I		Column-II
(A)	The value of the definite integral $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{x^4 + x^2 + 2}{(x^2+1)^2}\, dx$ equals	(P)	0
(B)	If $f(x) = x^{\frac{1}{\ln x}}$, then $f'(x) = 0$ for x equals	(Q)	1
(C)	The cosine of the angle of intersection of curves $f(x) = 2^x \ell nx$ and $g(x) = x^{2x} - 1$, is	(R)	2
(D)	If H is the number of horizontal tangents and V is the number of vertical tangents to the curve $y^3 - 3xy + 2 = 0$, then the value of (H + V) equals	(S)	2
		(T)	e

Subjective :

17. If the variable line $3x - 4y + k = 0$ lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circle, then the range of k is (a, b) where a, b $\in$ I. Find the value of (b – a).

18. Let $\sum_{r=1}^{n} \frac{r^4}{(2r-1)(2r+1)} = \frac{n^3}{A} + \frac{n^2}{B} + \frac{5n}{C} + \frac{f(n)}{D}$ (A, B, C, D $\in$ N)

where f(n) is the ratio of two linear polynomials such that $\lim_{n\to\infty} f(n) = \frac{1}{n}$

Find the value of (A + B + C + D).

19. If $\int_{0}^{\pi/4} \frac{\ln(\cot x)}{((\sin x)^{2009} + (\cos x)^{1009})^2} \cdot (\sin 2x)^{2008}\, dx = \frac{a^b \ln a}{c^2}$ (where a, b, c are in their lowest form) then find the value of (a + b + c).

PRACTICE TEST-14

Comprehension (Q.1 to Q.3)

Consider ,f g and h be three real valued function defined on R.

Let f(x) = sin 3x + cosa x, g(x) = cos 3x + sin x and h(x) = $f^2(x) + g^2(x)$

1. The length of a longest interval in which the function y = h(x) is increasing, is

(A) $\pi/8$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

2. General solution of the equation h(x) = 4, is

(A) $(4n + 1)\dfrac{\pi}{8}$ (B) $(8n + 1)\dfrac{\pi}{8}$ (C) $(2n + 1)\dfrac{\pi}{4}$ (D) $(7n + 1)\dfrac{\pi}{4}$

where $n \in I$

3. Number of point(s) where the graph of the two function, yh = f(x) and y = g(x) intersects in $[0, \pi]$, is

(A) 2 (B) 3 (C) 4 (D) 5

Single Choice Question :

4. Let a line ℓ through origin is tangent to the curve $y = x^3 + x + 16$. The slope of the line ℓ is

(A) 2 (B) 8 (C) 13 (D) 26

5. An ellipse has semi major axis of length 2 and semi minor axis of length 1. The distance between its foci is

(A) $2\sqrt{3}$ (B) 3 (C) $2\sqrt{2}$ (D) $\sqrt{3}$

6. The domain of definition of f(x) = $\log_{(x^2-x+1)}(2x^2 - 7x + 9)$ is

(A) R (B) R– {0} (C) R – {0, 1} (D) R – {1}

7. A coin that comes up head with probability p > 0 and tails with probability 1 – p > 0 independently on each fdlip, is flipped eight times. Suppose the probability of three heads and five tails is equal to $\dfrac{1}{25}$ of the probability of five heads and three tails. Let p = $\dfrac{m}{n}$, where m and n are relative prime positive integers The value of (m + n) equals

(A) 9 (B) 11 (C) 13 (D) 15

8. The value of the integral I = $\displaystyle\int_0^{2\alpha}\frac{\sqrt[4]{\sin(3\alpha - x)}}{\sqrt[4]{\sin(3\alpha - x)} + \sqrt[4]{\sin(\alpha + x)}}$ dx is

(A) $\alpha/2$ (B) 0 (C) 2α (D) α

9. Let f, g and h are differentiable function such that g(x) = f(x) – x and h(x) = f(x) – x^3 are both strictly increasing functions, then the function F(x) = f(x) – $\dfrac{\sqrt{3}x^2}{2}$ is

(A) Strictly increasing $\forall\ x \in R$

(B) Strictly decreasing $\forall\ x \in R$

(C) Strictly decreasing on $\left(-\infty, \dfrac{1}{\sqrt{3}}\right)$ and strictly increasing on $\left(\dfrac{1}{\sqrt{3}}, \infty\right)$

(D) Strictly increasing on $\left(-\infty, \dfrac{1}{\sqrt{3}}\right)$ and strictly decreasing on $\left(\dfrac{1}{\sqrt{3}}, \infty\right)$

10. Consider the cubic $x^3 - x^2 + 3x + 4 = 0$ where a, b and c are its roots and let $w = \tan^{-1} a + \tan^{-1} b + \tan^{-1} c$. If the absolute value of $\sec \omega = \frac{\sqrt{m}}{n}$ where m and n are prime numbers, then the value of (m + n) equals

(A) 13 (B) 31 (C) 29 (D) 63

11. The value of $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{n-k}{n^2} \cos \frac{4k}{n}$ equals

(A) $\frac{1}{4}\sin 4 + \frac{1}{16}\cos 4 - \frac{1}{16}$ (B) $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

(C) $\frac{1}{16}(1 - \sin 4)$ (D) $\frac{1}{16}(1 - \cos 4)$

12. Number of four digit numbers with all digits different and containing the digit 7 is

(A) 2016 (B) 1828 (C) 1848 (D 1884

13. The value of $\sum\limits_{m=0}^{2009}\sum\limits_{n=0}^{m}\binom{2009}{m}\binom{m}{n}$ equals

(A) 2009 (B) 2^{2009} (C) 3^{2009} (D) 3^{2010}

14. If $\cos^{-1}\frac{x}{a} - \sin^{-1}\frac{y}{b} = \theta$, $(a, b \neq 0)$ then the maximum value of $b^2x^2 + a^2y^2 + 2ab\,xy \sin\theta$ euqals

(A) ab (B) $(a + b)^2$ (C) $2(a + b)^2$ (D) $a^2 b^2$

15. In ΔABC, the bisector of the angle A meets the side BC at D and the circumscribed circle at E, the DE equals

(A) $\frac{a^2 \sec\frac{A}{2}}{2(b+c)}$ (B) $\frac{a^2 \sin\frac{A}{2}}{2(b+c)}$ (C) $\frac{a^2 \cos\frac{A}{2}}{2(b+c)}$ (D) $\frac{a^2 \operatorname{cosec}\frac{A}{2}}{2(b+c)}$

16. Area enclosed by the curve $y = (x^2 + 2x)\,e^{-x}$ and the positive x-axis is

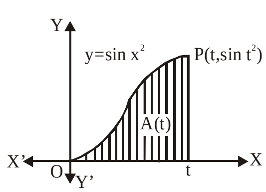
(A) 1 (B) 2 (C) 4 (D) 6

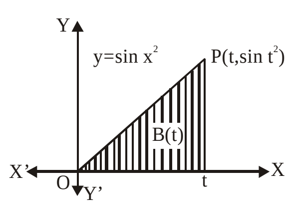
17. Equation of the circle which cuts the circle $x^2 + y^2 + 2x + 4y - 4 = 0$ and the lines $xy - 2x - y + 2 = 0$ orthogonally, is

(A) $x^2 + y^2 - 2x - 4y - 6 = 0$ (B) $x^2 + y^2 - 2x - 4y + 6 = 0$

(C) $x^2 + y^2 - 2x - 4y + 12 = 0$ (D) Not possible to determine

18. The figure shows two regions in the first quadrant. A(t) is the area under the curve $y = \sin x^2$ from 0 to t and B(t) is the area of the triangle with vertices O, P and M(t, 0). $\lim\limits_{t\to 0}\frac{A(t)}{B(t)}$ equals





(A) 1/3 (B) 1/2 (C) 3/5 (D) 2/4

19. A curve in the first quadrant is such that the area of the triangle formed in the first quadrant by the x-axis, a tangent ot the curve at any of its point P and radius vector of the point P is 2 sq. units. If the curve passes through (2, 1), then the equation of the curve can be

(A) $x^2 - y^2 = 3$ (B) $xy = 2$ (C) $x^2 = 4y$ (D) $8y = x^3$

20. The acute angle between the line $3x - 4y = 5$ and the circle $x^2 + y^2 - 4x + 2y - 4 = 0$ is θ, then $\sin\theta$ equals

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{1}{3}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{2\sqrt{2}}{3}$

21. Let $g : [0, \infty) \to R$ be a continuous and strictly increasing function such that $f^3(x) = \int_0^x t\, f^2(r)\, dt$, $\forall\ x > 0$.

The area enclosed by $y = f(x)$, the x-axis and the ordinate at $x = 3$, is

(A) 1 (B) 3/2 (C) 2 (D) 3

22. Let S(t) be the area of the ΔOAB with O(0, 0, 0), A(2, 2, 1) and B(t, 1, t + 1). The value of the definite integral $\int_1^e (S(t))^2\ \ell n\, t\, dt$, is equal to

(A) $\frac{2e^3 + 5}{2}$ (B) $\frac{e^3 + 5}{2}$ (C) $\frac{2e^3 + 15}{2}$ (D) $\frac{e^3 + 15}{2}$

One or more than one correct :

23. Let $\ell_1 = \lim_{x \to \infty} \sqrt{\frac{x - \cos^2 x}{x + \sin x}}$ and $\ell_2 = \lim_{h \to 0^+} \int_{-1}^{1} \frac{h\,dx}{h^2 + x^2}$. Then

(A) Both ℓ_1 and ℓ_2 are less than 22/7
(B) One of the tow limits is rational and other irrational
(C) $\ell_2 > \ell_1$
(D) ℓ_2 is greater than 3 times of ℓ_1

24. Let $f : R \to R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where $\{x\}$ is fractional part function. Then which of the following is/are correct ?

(A) f is many one but not even function
(B) Range of f contains two prime numbers
(C) f is aperiodic
(D) Graph of f does not lie below x-axis

25. Which of the following is/are True ?
The circles $x^2 + y^2 - 6x - 6y + 9 = 0$ and $x^2 + y^2 + 6x + 9 = 0$ are such that

(A) They do not intersect
(B) They touch each other
(C) Their exterior common tangents are parallel
(D) Their interior common tangents are perpendicular

26. The first term of an infinite geometric series is 21. The second term and the sum of the series are both positive integers. The possible value(s) of the second term can be

(A) 12 (B) 14 (C) 18 (D) 20

27. In an acute triangle ABC, if the coordinates of orthocentre 'H' are (4, b), centroid 'G' are (b, 2b – 8) and circumcentre 'S' are (–4, 8), then 'b' can not be

(A) 4 (B) 8 (C) 12 (D) – 12

28. A line L passing through the point P(1, 4, 3), is perpendicular to both the lines

$$\frac{x-1}{2} + \frac{y+3}{1} = \frac{z-2}{4} \text{ and } \frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$$

If the positive vector of point Q on L is (a_1, a_2, a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be

(A) 16 (B) 15 (C) 2 (D) 1

29. A certain coin lands head with probability p. Let Q denote the probability that when the coin is tossed four times the number of heads obtained is even. Then

(A) There is no value of p, if Q = 1/4

(B) There is exactly one value of p is Q = 3/4

(C) There are exactly two values of p is Q = 3/5

(D) There are exactly four values of p is Q / 4/5

30. The position vectors of the vertices A, B and C of a tetrahedron are (1, 1, 1), (1, 0, 0) and (3, 0, 0) respectively. The altitude from the vertex D to the opposite face ABC meets the median line through A of the ΔABC at a point E. If the length of side AD is 4 and volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$ then the correct statement(s) is/are

(A) The altitude from the vertex D is 2

(B) There is exactly one position for point E

(C) There can be two position for the point E

(D) Vector $\hat{j}-\hat{k}$ is normal to the plane ABC

PRACTICE TEST-15

Comprehension (Q.1 to Q.3)

Consider f, g and h be three real valued differentiable functions defined on R.
Let $g(x) = x^3 + g''(1)\, x^2 + (3g'(1) - g''(1) - 1)\, x + 3g'(1)$, $f(x) = x\, g(x) - 12x + 1$ and $f(x) = (h(x))^2$ where $h(0) = 1$.

1. The function $y = f(x)$ has
(A) Exactly one local minimum and no local maxima
(B) Exactly one local maxima and no local minima
(C) Exactly one local maxima and two local minima
(D) Exactly two local maxima and one local minima

2. Which of the following is/are true for the function $y = g(x)$?

(A) g(x) monotonically decreases in $\left(-\infty, 2-\frac{1}{\sqrt{3}}\right) \cup \left(2+\frac{1}{\sqrt{3}}, \infty\right)$

(B) g(x) monotonically increases in $\left(2-\frac{1}{\sqrt{3}}, 2+\frac{1}{\sqrt{3}}\right)$

(C) Three exists exactly one trangent to $y = g(x)$ which is parallel to the chord joining the points (1, g) and (3, g(3))
(D) There exists exactly two distinct Lagrange's mean value in (0, 4) for the function $y = g(x)$

3. Which one of the following two distinct Lagrange's mean value in (0, 4) for the function $y = g(x)$
(A) Exactly one critical point
(B) No point of inflection
(C) Exactly one real zero in (0, 3)
(D) Exactly one tangent parallel to x-axis

Assertion & Reason :

4. Consider a differentiable function $y = f(x)$ which satisfies $f(x) = \int_0^x f(t)\sin t - \sin(t-x))\, dt$

Statement-1 : The differential equation corresponding to $y = f(x)$ is a first order linear different equation.

Statement-2 : The differential equation corresponding to $y = f(x)$ is of degree one.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

5. **Statement-1 :** Sum of 2009 consecutive netural numbers is divisible by 2009.

Statement-2 : Sum of n consecutive natural numbers is always divisible by n for $n > 4$, $n \in N$.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

6. **Statement-1 :** Let A = {1, 2, 3, 4, 5, 6}. If a, b, c $\in$ A, then the probability that

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{3/x} = 6 \text{ is } \frac{1}{16}$$

Statement-2 : $\lim_{x\to 0}\left(\frac{a_1^x+a_2^x+a_3^x}{3}\right)^{3/x} = a_1\, a_2\, a_3$, where $a_1 > 0$, $a_2 > 0$, $a_3 > 0$.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

7. **Statement-1 :** Let C_1 (0, 0) and C_2 (2, 2) be centres of two circles and L : x + y – 2 = 0 is their common chord. If length of common chord is equal to $2\sqrt{2}$, then both circles intersect orthogonally.

Statement-2 : Two circles will be orthogonal if their centres are mirror images of each other in their common chord and distance between centres is equal to lenglth of common chord.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

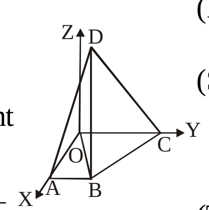
Match the Column :

8. Consider a square matrix A of order 2 which has its elements as 0, 1, 2 and 4. Let N denote the number of such matrices.

	Column-I		**Column-II**
(A)	Possible non-negative value of det(A) is	(P)	2
(B)	Sum of values of determinants corresponding to N matrices is	(Q)	4
(C)	If absolute value of (det(A)) is least, then possible value of |adj(adj(adj A))|	(R)	– 2
(D)	If det (a) is algebratically least, then possible value of det(4 A^{-1}) is	(S)	0
		(T)	8

9.

	Column-I		**Column-II**
(A)	Possible integral value(s) of k for which the point M(0, k) lies on or inside the triangle formed by the lines y + 3x + 2 = 0, 3y – 2x – 5 = 0 and 4y + x – 14 = 0.	(P)	0
(B)	If $\hat{a}, \hat{b}$ and $\hat{c}$ are non coplanar vectors, then the vectors $\vec{V}_1 = \hat{a} + 2\hat{b} + 3\hat{c}$, $\vec{V}_2 = \lambda + \hat{b} + 4\hat{c}$ and $\vec{V}_3 = (2\lambda - 1)\hat{c}$ where λ is a scalar, can be non coplanr, for λ equals	(Q)	1
(C)	The distance of the z-axis from the image of the point A(2, –3, 3) in the plane x – 2y – z + 1 = 0, is	(R)	2
(D)	The figure shows a pyramid DOABC (where O is the origin) with a square base whose sides are 1 unit log. The pyramid's height is also 1 unit and the point D stands directly above the mid point of the diagonal OB. If the angle between $\overrightarrow{OB}$ and $\overrightarrow{OD}$ is $\tan^{-1}\sqrt{K}$, then the equal to	(S)	3
		(T)	4



Subjective :

10. Consider the circle W = $x^2 + y^2 = 81$. Let AB be a diameter of circle W. AB is extended through A to C. Point T lies on W so that line CT is tangent to W. Point P is the foot of the perpendicular from A to the line CT. Find the maximum value of $(BP)^2$.

11. Given y(0) = 2000 and $\frac{dy}{dx} = 32000 - 20y^2$, then find the value of $\lim_{x\to\infty} y(x)$.

12. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors, then find the maximum value of $|2\hat{a} - 3\hat{b}|^2 + |2\hat{b} - 3\hat{c}|^2 + |2\hat{c} - 3\hat{a}|^2$.

13. The sides of a triangle have the combined equation $x^2 - 3y^2 - 2xy + 8y - 4 = 0$. The third side, which is variable always passes through the point (–5, –1). If the range of value of the slope of the third line so that the origin is an interior point of the triangle, line in the interval (a, b) then find $\left(a + \frac{1}{b^2}\right)$.

14. A pair of students is selected at random from a probability class. The probability that the pair selected will consists of one male and one female student is $\frac{10}{19}$. Find the maximum number of students the class can contain.

15. Let $I_n = \int_{-1}^{1} |x| \left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2n}}{2n}\right) dx$. If $\lim_{n\to\infty} I_n$ can be expressed as rational $\frac{p}{q}$ in the lowest form, then find the value of $pq\,(p^3 + q^2)$.

ANSWER KEY

TEST-1

1. B **2.** C **3.** A **4.** C **5.** D **6.** D **7.** A–S ; B–P ; C–Q ; D–R **8.** A–R ; B–P ; C–S ; D–Q
9. C **10.** A **11.** D **12.** D **13.** B **14.** B **15.** AB **16.** ABCD **17.** ABCD **18.** AC

TEST-2

1. D **2.** D **3.** D **4.** C **5.** D **6.** A **7.** BD **8.** ABD **9.** AB **10.** BC **11.** D **12.** A **13.** B **14.** D
15. D **16.** B **17.** A–P,R,S ; B–Q,R,S ; C–R,S ; D–P,R,S **18.** A–P ; B–PQ ; C–Q ; D–P

TEST-3

1. A **2.** C **3.** B **4.** A **5.** AB **6.** BCD **7.** ABC **8.** ABC **9.** A–Q ; B–R ; C–P ; D–S
10. A–PQ ; B–R,S ; C–S,T ; D–P,Q **11.** 4 **12.** 6 **13.** 1 **14.** 6 **15.** 4 **16.** 9 **17.** 3 **18.** 1

TEST-4

1. D **2.** A **3.** A **4.** C **5.** AB **6.** C **7.** BC **8.** AB **9.** B **10.** A **11.** B **12.** A **13.** C **14.** A
15. A–S ; B–R ; C–P ; D–Q **16.** A–S ; B–R ; C–P ; D–P,Q,R,S

TEST-5

1. A **2.** A **3.** B **4.** A **5.** A **6.** C **7.** ABCD **8.** AD **9.** CD **10.** AC **11.** A–P ; B–Q ; C–R ; D-S
12. A–R ; B–P ; C–S; D–Q **13.** C **14.** A **15.** D **16.** B **17.** C **18.** A

TEST-6

1. C **2.** C **3.** C **4.** D **5.** A **6.** A **7.** A–Q ; B–S ; C–Q ; D–S **8.** A–R ; B–P ; C–S ; D–Q
9. 15 **10.** 2 **11.** 50 **12.** 729 **13.** 210 **14.** 1 **15.** 9

TEST-7

1. ABCD **2.** BC **3.** ABC **4.** AB **5.** ABCD **6.** BD **7.** ACD **8.** D **9.** C **10.** B **11.** A **12.** B
13. B **14.** A–Q ; B–R ; C–P ; D–S **15.** A-R ; B–P ; C–Q ; D–P

TEST-8

1. A **2.** B **3.** D **4.** D **5.** A **6.** B **7.** A–P ; B–Q ; C–Q ; D–Q **8.** A–R ; B–P ; C–A ; D–PQ
9. 0 **10.** 132 **11.** 2 **12.** 2

TEST-9

1. D **2.** A **3.** C **4.** B **5.** B **6.** A **7.** D **8.** A **9.** C **10.** AC **11.** ACD **12.** ABD
13. A–P,Q,R,S ; B–R ; C–R,S **14.** A–R; B–S ; C–Q ; D–Q

TEST-10

1. B **2.** D **3.** A **4.** C **5.** D **6.** A **7.** B **8.** C **9.** C **10.** B **11.** D **12.** B **13.** A **14.** B
15. A **16.** A **17.** C **18.** D **19.** B **20.** D **21.** C **22.** B **23.** ABC **24.** BCD **25.** AC **26.** BD
27. AB **28.** AB **29.** ABC **30.** CD

TEST-11

1. B **2.** A **3.** C **4.** A **5.** D **6.** A **7.** B **8.** A–QT ; B–P ; C–Q ; D–R
9. A–P,Q,R,T, ; B–Q,S ; C–P,Q,R,T ; D–P **10.** 1125 **11.** 16 **12.** 12 **13.** 4 **14.** 13 **15.** 8

TEST-12

1. C **2.** D **3.** B **4.** A **5.** C **6.** C **7.** C **8.** A **9.** D **10.** C **11.** BCD **12.** AWBCD **13.** BD
14. A–R ; B–S ; C–P **15.** A–Q ; B–R ; C–P **16.** 65 **17.** 5

TEST-13

1. A **2.** A **3.** A **4.** A **5.** A **6.** B **7.** A **8.** D **9.** B **10.** D **11.** A **12.** D **13.** ABCD

14. ACD **15.** BD **16.** A–R ; B–R,S,T ; C–Q ; D–Q

TEST-14

1. B **2.** A **3.** C **4.** C **5.** A **6.** C **7.** B **8.** D **9.** A **10.** B **11.** D **12.** C **13.** C **14.** F
15. A **16.** C **17.** A **18.** D **19.** B **20.** D **21.** B **22.** B **23.** ABCD **24.** ABD **25.** ACD **26.** ABCD
27. ABCD **28.** BD

TEST-15

1. C **2.** D **3.** C **4.** B **5.** C **6.** D **7.** A **8.** A–P,Q,T ; B–S ; C–P,R ; D–R
9. A–R,S ; B–Q,R,S,T ; C–Q ; D–R **10.** 432 **11.** 40 **12.** 57 **13.** 24 **14.** 20 **15.** 186