VKR Classes

TIME BOUND TESTS 1-7

Target JEE ADVANCED

For Class XI
PRACTICE TEST-1

Single Choice Question:

1. The smallest integer greater than \[ \frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi}, \] is
   (A) 1         (B) 2         (C) 3         (D) 4

2. If \( x = \cos \alpha + \cos \beta - \cos (\alpha + \beta) \) and \( y = 4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \left( \frac{\alpha + \beta}{2} \right), \) then \( (x - y) \) equals
   (A) 0         (B) 1         (C) –1        (D) –2

3. If \( \alpha, \beta \) are the roots of the quadratic equation \( x^2 - (3 + 2\log \sqrt{3} - 3\log \sqrt{2}) x - 2 (3\log \sqrt{2} - 2\log \sqrt{3}) = 0, \) then the value of \( \alpha^2 + \alpha \beta + \beta^2 \) is equal to
   (A) 3         (B) 5         (C) 7         (D) 11

4. Let \( f(x) = 3ax^2 - 4bx + c \) (a, b, c \( \in \mathbb{R} \), a \( \neq 0 \)) where a, b, c are in A.P. Then the equation \( f(x) = 0 \) has
   (A) no real solution        (B) two unequal real roots
   (C) sum of roots always negative (D) product of roots always positive

5. Let the vertices of an equilateral \( \triangle ABC \) be \((1, 1), (-1, -1)\) and \((a, b)\). Then consider the following four statements.
   I. \( a^2 + b^2 \) must be equal to 6
   II. \( a + b \) must be equal to zero
   III. \( a + b \) can be equal to \( 2\sqrt{3} \)
   IV. length of its median is \( \sqrt{6} \)

6. Let \( f(\theta) = \frac{1}{2} + \frac{2}{3} \csc^2 \theta + \frac{3}{8} \sec^2 \theta. \) The least value of \( f(\theta) \) for all permissible value of \( \theta, \) is
   (A) \( \frac{31}{12} \)        (B) \( \frac{61}{48} \)        (C) \( \frac{61}{25} \)        (D) \( \frac{61}{24} \)

7. Let A(2, –3) and B(–2, 1) be vertices of a \( \triangle ABC. \) If the centroid of \( \triangle ABC \) moves on the line \( 2x + 3y = 1, \) then the locus of the vertex C is
   (A) \( 2x + 3y = 9 \)        (B) \( 2x - 3y = 7 \)        (C) \( 3x + 2y = 5 \)        (D) \( 3x - 2y = 3 \)

8. The value of the expression \( \frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} \) equals
   (A) \( \frac{1}{2} (\tan 9x - \tan x) \)        (B) \( \frac{1}{2} (\tan 9x - \tan 3x) \)
   (C) \( \frac{1}{2} (\tan 27x - \tan x) \)        (D) \( \frac{1}{2} (\tan 27x - \tan 3x) \)

9. If the angles subtended by the sides of a triangle at orthocentre and incentre are equal, then the triangle is
   (A) Scalene        (B) Isosceles but not equilateral
   (C) Equilateral    (D) Obtuse angled

10. Let there exist a unique point \( P \) inside a \( \triangle ABC \) such that \( \angle PAB = \angle PBC = \angle PCA = \alpha. \)
    If \( PA = x, PB = y, PC = z, \Delta = \text{area of } \triangle ABC \) and \( a, b, c \) are the sides opposite to \( \alpha \) angle A, B, C respectively. then \( \tan \alpha \) is equal to
    (A) \( \frac{a^2 + b^2 + c^2}{4\Delta} \)        (B) \( \frac{a^2 + b^2 + c^2}{2\Delta} \)
    (C) \( \frac{2\Delta}{a^2 + b^2 + c^2} \)        (D) \( \frac{4\Delta}{a^2 + b^2 + c^2} \)
11. Let \( \alpha, \beta, \gamma \) are the roots of the cubic equation \( a_0 x^3 + 3ax^2 + 3a_2 x + a_3 = 0 \) \( (a_0 \neq 0) \). Then the value of \( (\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2 \) equals

(A) \( \frac{18a_1^2 - a_0a_2}{a_0^2} \)  
(B) \( \frac{18a_2^2 + a_0a_1}{a_0^2} \)  
(C) \( \frac{18a_3^2 + a_1a_2}{a_0^2} \)  
(D) \( \frac{18a_1^2 - a_0a_3}{a_0^2} \)

12. If \( \frac{1 + 3 + 5 + \ldots \text{upto n terms}}{4 + 7 + 10 + \ldots \text{upto n terms}} = \frac{20}{7 \log_{10} x} \)
and \( n = \log_{10} x + \log_{10} x^{1/2} + \log_{10} x^{1/4} + \log_{10} x^{1/8} + \ldots + \infty \), then \( x \) equal to

(A) \( 10^3 \)  
(B) \( 10^5 \)  
(C) \( 10^6 \)  
(D) \( 10^7 \)

13. The value of \( \sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{n}{5^n} \right) \) equals

(A) \( \frac{5}{12} \)  
(B) \( \frac{5}{24} \)  
(C) \( \frac{5}{36} \)  
(D) \( \frac{5}{16} \)

14. In a \( \Delta ABC \) with usual notations, if \( r = 1, r_1 = 7 \) and \( R = 3 \), then the \( \Delta ABC \) is

(A) equilateral  
(B) acute angled which is not equilateral  
(C) obtuse angled  
(D) right angled

One or more than one correct :

15. 

\[
\begin{vmatrix}
1 & \sin \theta & 1 \\
-\sin \theta & 1 & \sin \theta \\
-1 & -\sin \theta & 1 \\
\end{vmatrix}
\]

(A) can not equals three for atleast one value of \( \theta \in R \)  
(B) is zero for some value of \( \theta \in R \)  
(C) lies in \([2, 4]\)  
(D) lies in \([-1, 1]\)

Subjective :

16. Let the lengths of the altitudes drawn from the vertices of a \( \Delta ABC \) to the opposite sides are 2, 2 and 3. If the area of \( \Delta ABC \) is \( \Delta \), then find the value of \( 2 \sqrt{\frac{\Delta}{2}} \).

17. If the expression \( \cos^2 \frac{\pi}{11} + \cos^2 \frac{2\pi}{11} + \cos^2 \frac{3\pi}{11} + \cos^2 \frac{4\pi}{11} + \cos^2 \frac{5\pi}{11} \) has the value equal to \( \frac{p}{q} \) in their lowest has integral roots.

18. Find the sum of all positive integral value(s) of \( n \), \( n \in [1, 300] \) for which the quadratic equation \( x^2 - 3x - n = 0 \) has integral roots.

19. If \( (x_1 - x_2)^2 + (y_1 - y_2)^2 = a^2 \)  
\( (x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2 \)  
and \( (x_3 - x_1)^2 + (y_3 - y_1)^2 = c^2 \)  
then \( \lambda \cdot \begin{vmatrix}
x_1 & y_1 & 1 \\
x_2 & y_2 & 1 \\
x_3 & y_3 & 1 \\
\end{vmatrix}^2 = (a+b+c)(b+c-a)(c+a-b)(a+b-c) \). Find the value of \( \lambda \).

20. If \( \begin{vmatrix}
\sin 30 & -1 & 1 \\
\cos 20 & 4 & 3 \\
2 & 7 & 7 \\
\end{vmatrix}^2 = 0 \), then find the number of values of \( j \) in \([0, 2\pi]\).
Consider a \( \triangle ABC \) whose sides BC, CA and AB are represented by the straight lines \( x - 2y + 5 = 0 \), \( x + y + 2 = 0 \) and \( 8x - y - 20 = 0 \) respectively.

1. The area of \( \triangle ABC \) equals
   
   \[
   \begin{align*}
   (A) & \quad \frac{41}{2} \\
   (B) & \quad \frac{43}{2} \\
   (C) & \quad \frac{45}{2} \\
   (D) & \quad \frac{47}{2}
   \end{align*}
   \]

2. If AD be the median of the \( \triangle ABC \) then the equation of the straight line passing through (2, –1) and parallel to AD is
   
   \[
   \begin{align*}
   (A) & \quad 4x - 3y - 11 = 0 \\
   (B) & \quad 13x - 4y - 30 = 0 \\
   (C) & \quad 4x + 13y + 5 = 0 \\
   (D) & \quad 13x + 4y - 22 = 0
   \end{align*}
   \]

3. The orthocentre of the \( \triangle ABC \) is
   
   \[
   \begin{align*}
   (A) & \quad (-3, 1) \\
   (B) & \quad \left(-\frac{1}{3}, \frac{2}{3}\right) \\
   (C) & \quad (-2, 4) \\
   (D) & \quad \left(-\frac{2}{3}, \frac{4}{3}\right)
   \end{align*}
   \]

Assertion & Reason :

4. \textbf{Statement-1} : If \( a + b + c > 0 \) and \( a < 0 < b < c \), then both roots of the quadratic equation
   \[
   a(x - b) (x - c) + b (x - c) (x - a) + c (x - a) (x - b) = 0
   \]
   are real and unequal.

   \textbf{because}

   \textbf{Statement-2} : If both roots of the quadratic equation \( px^2 + qx + r = 0 \) are of opposite sign then product
   of roots is negative and sum of roots is positive.

   \[
   (A) \text{ Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1} \\
   (B) \text{ Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1} \\
   (C) \text{ Statement-1 is true, Statement-2 is false} \\
   (D) \text{ Statement-1 is false, Statement-2 is true.}
   \]

5. \( \text{Let } 0 < \alpha, \beta, \gamma < \frac{\pi}{2} \)

   \textbf{Statement-1} : If \( \tan^3 \alpha, \tan^3 \beta, \tan^3 \gamma \) are the roots of the cubic equation \( x^3 - 6x^2 + kx - 8 = 0 \), then \( \tan \alpha = \tan \beta = \tan \gamma \).

   \textbf{because}

   \textbf{Statement-2} : If \( a^3 + b^3 + c^3 = 3abc \) and \( a, b, c \) are positive numbers then \( a = b = c \).

   \[
   (A) \text{ Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1} \\
   (B) \text{ Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1} \\
   (C) \text{ Statement-1 is true, Statement-2 is false} \\
   (D) \text{ Statement-1 is false, Statement-2 is true.}
   \]

6. \textbf{Statement-1} : In any \( \triangle ABC \), maximum value of \( r_1 + r_2 + r_3 = \frac{9R}{2} \).

   \textbf{because}

   \textbf{Statement-2} : In any \( \triangle ABC \), \( R \geq 2r \).

   \[
   (A) \text{ Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1} \\
   (B) \text{ Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1} \\
   (C) \text{ Statement-1 is true, Statement-2 is false} \\
   (D) \text{ Statement-1 is false, Statement-2 is true.}
   \]
7. **Statement-1**: If the sides of the \( \Delta ABC \) are along the lines \( L_1, L_2 \) and \( L_3 \) then there is only one point in the plane of \( \Delta ABC \) which is equidistant from the lines \( L_1, L_2 \) and \( L_3 \).

**because**

**Statement-2**: Incentre of the \( \Delta ABC \) is equidistant from the lines \( L_1, L_2 \) and \( L_3 \).
(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

8. Let \( l_1, l_2, l_3 \) be the lengths of the internal bisectors of angles \( A, B, C \) respectively of a \( \Delta ABC \).

**Statement-1**:
\[
\frac{\cos A}{2} \cdot l_1 + \frac{\cos B}{2} \cdot l_2 + \frac{\cos C}{2} \cdot l_3 = 2 \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)
\]

**because**

**Statement-2**:
\[
l_1^2 = bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right], l_2^2 = ca \left[ 1 - \left( \frac{b}{c+a} \right)^2 \right], l_3^2 = ab \left[ 1 - \left( \frac{c}{a+b} \right)^2 \right]
\]

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

**One or more than one correct:**

9. Let \( P(x) = ax^2 + bx + c, Q(x) = ax^2 + cx + b \) and \( R(x) = ax^2 + bcx + b^3 + c^3 - 4abc \) where \( a, b, c \in \mathbb{R} \) and \( a \neq 0 \). The equation \( R(x) = 0 \) will have non real roots if
(A) \( P(x) = 0 \) has distinct real roots and \( Q(x) = 0 \) has non-real roots
(B) \( P(x) = 0 \) has non-real roots \( Q(x) = 0 \) has distinct real roots
(C) Both \( P(x) = 0 \) and \( Q(x) = 0 \) has non-real roots
(D) Both \( P(x) = 0 \) and \( Q(x) = 0 \) have distinct real roots

10. Let \( a, b, c \) be unequal real numbers. If \( a, b, c \) are in G.P. and \( a + b + c = bx \), aten ‘\( x \)’ can not be equal to
(A) – 1    (B) 0    (C) 2    (D) 3

11. If \( 1 + \log_5 (x^2 + 1) \geq \log_5 (ax^2 + 4x + a), \quad \forall x \in \mathbb{R} \) then ‘\( a \)’ can be equal to
(A) 3    (B) 5/2    (C) 2    (D) 3/2

12. The expression \( (\alpha \tan \gamma + \beta \cot \gamma) (\alpha \cot \gamma + \beta \tan \gamma) - 4\alpha \beta \cot^2 2\gamma \) is
(A) independent of \( \alpha, \beta \)    (B) independent of \( \gamma \)
(C) dependent on \( \gamma \)    (D) dependent on \( \alpha, \beta \)

13. In a \( \Delta AEX \), \( T \) is the mid point of \( XE \), and \( P \) is the mid point of \( ET \). If the \( \Delta APE \) is equilateral of side length equal to unity then which of the following alternative(s) is/are correct ?
(A) \( AX = \sqrt{13} \)    (B) \( \angle EAT = 90^\circ \)    (C) \( \cos \angle XAE = \frac{-1}{\sqrt{3}} \)    (D) \( AT = \frac{1}{\sqrt{3}} \)
Match the Column:

14. Match the Column-I with Column-II:

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) If sum of the solution of the equation ( \cot x + \csc x + \sec x = \tan x ) in ([0, 2\pi]) is (\frac{k\pi}{2}), then the value of (k) is greater than (P) 1</td>
<td></td>
</tr>
<tr>
<td>(B) If (x = 111\ldots1) (20 digits), (y = 333\ldots3) (10 digits) and (z = 222\ldots2) (10 digits), then (\frac{x - y^2}{2}) equals (Q) 2</td>
<td></td>
</tr>
<tr>
<td>(C) Possible integral values of ‘(a)’ for which (a^2 - 6 \sin x - 5 a \leq 0, \forall ; x \in \mathbb{R}), is (S) 5</td>
<td></td>
</tr>
</tbody>
</table>

15. Let \(P\) be an interior point of \(\triangle ABC\).

Match the correct entries for the ratios of the Area of \(\triangle PBC\) : Area of \(\triangle PCA\) : Area of \(\triangle PAB\) depending on the position of the point \(P\) w.r.t. \(\triangle ABC\). Marks will be given only if all the entries of the column-I are correctly matched.

<table>
<thead>
<tr>
<th>Column-I</th>
<th>Column-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) If (P) is centroid (G)</td>
<td>(P) (\tan A : \tan B : \tan C)</td>
</tr>
<tr>
<td>(B) If (P) is incentre (I)</td>
<td>(Q) (\sin 2A : \sin 2B : \sin 2C)</td>
</tr>
<tr>
<td>(C) If (P) is orthocentre (H)</td>
<td>(C) (\sin A : \sin B : \sin C)</td>
</tr>
<tr>
<td>(D) If (P) is circumcentre (S)</td>
<td>(S) 1 : 1 : 1</td>
</tr>
<tr>
<td></td>
<td>(T) (\cos A : \cos B : \cos C)</td>
</tr>
</tbody>
</table>
PRACTICE TEST-3

Single Choice Question:

1. If \( \sin x + \sin^2 x + \sin^3 x = 1 \), then the value of \( \cos^6 x - 4 \cos^4 x + 8 \cos^2 x \) equals
   (A) 2   (B) 3   (C) 4   (D) 5

2. The number of straight lines equidistant from three non collinear point in the plane of the points equals
   (A) 0   (B) 1   (C) 2   (D) 3

3. A variable straight line passes through the point of intersection of the lines \( x + 2y = 1 \) and \( 2x - y = 1 \) and meets the co-ordinate axes in A and B. The locus of the middle point of AB is
   (A) 5xy = 3y + x   (B) 10xy = 3x + y   (C) 10xy = x + 3y   (D) 5xy = 3x + y

4. The points \((-a, -b), (0, 0), (a, b), (a^2, ab), (ab^2)\), \((ab^2, 0)\)
   (A) are collinear   (B) are the vertices of a parallelogram   (C) are the vertices of a rectangle   (D) lie on a circle

5. Let \( D, E \) be the roots of \( ax^2 + bx + c = 0 \) \((a \neq 0)\) and \( J, G \) be the roots of \( px^2 + qx + r = 0 \) \((p \neq 0)\), and \( D_1, D_2 \)
   be the respective discriminants of these equations. If \( D, E, J, G \) are in A.P., then \( D_1 : D_2 \) equals
   (A) \( \frac{2a^2}{p^2} \)   (B) \( \frac{2a^2}{b^2} \)   (C) \( \frac{2b^2}{q^2} \)   (D) \( \frac{2c^2}{r^2} \)

6. If the points \((1, 2)\) and \((3, 4)\) lies on the same side of the straight line \( 3x - 5y = a \), then
   (A) \(-11 < a < -7\)   (B) \(a = -11\)   (C) \(a = -7\)   (D) \(a < -11\) or \(a > -7\)

Paragraph for question nos. 7 to 9

In \( \triangle ABC \) as shown, \( XX_1 = d_1; XX_2 = d_2; XX_3 = d_3 \) and X is the centre of the circumscribed circle
around the \( \triangle ABC \). a, b and c as usual are sides BC, CA and AB respectively.

7. If \( \lambda \left( \frac{a}{d_1} + \frac{b}{d_2} + \frac{c}{d_3} \right) = \frac{abc}{d_1d_2d_3} \), then the value of ‘\( \lambda \)’ is equal to
   (A) 1   (B) 2   (C) 4   (D) 8

8. If R is the radius of the circumcircle of the \( \triangle ABC \) and \( a(d_2 + d_3) + b(d_3 + d_1) + c(d_1 + d_2) = kR(a + b + c) \)
then the value of ‘\( k \)’ is
   (A) 1   (B) 1/2   (C) 1/3   (D) 2

9. Let \( h_a, h_b \) and \( h_c \) are the altitudes of the \( \triangle ABC \) from the angular points A, B and C respectively
   If \( a^2 + b^2 + c^2 = t(h_a d_1 + h_b d_2 + h_c d_3) \) then ‘\( t \)’ equals
   (A) 1   (B) 2   (C) 3   (D) 4

One or more than one correct:

10. The equation of the straight line which passes through the point of intersection of the lines
    \( 3x - 4y + 1 = 0 \) and \( 5x + y - 1 = 0 \) and cuts off equal intercept on coordinate axes, is
    (A) \( 23x - 23y + 5 = 0 \)   (B) \( 23x - 23y - 11 = 0 \)   (C) \( 23x + 23y + 5 = 0 \)   (D) \( 23x + 23y - 11 = 0 \)

11. In \( \triangle ABC \), if \( \cos A + \cos B = 4 \sin^2 \frac{C}{2} \), then which of the following hold(s) good?
    (A) \( \cot \frac{A}{2} \cot \frac{B}{2} = 2 \)   (B) \( \cot \frac{A}{2} \cot \frac{B}{2} = 3 \)   (C) \( a, b \) are in A.P.   (D) \( a, b, c \) are in G.P.
12. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. The equation of the third side can be
(A) $x + 3y + 29 = 0$  (B) $x - 3y = 31$  (C) $3x - y = 13$  (D) $3x + y + 7 = 0$

Match the Column

13. Column-I Column-II

(A) The value of the $\sum_{n=0}^{\infty} \frac{2n+3}{3^n}$, is equal to (P) 2
(B) Number of value(s) of k for which atleast one root of the equation $(k^3 - 3k^2 - 4k + 12)x^2 + (k^4 - 13k^2 + 36)x + (k^2 - 7k + 10) = 0$ (A) 4
(C) In $\triangle ABC$, if $\cos A \cos B + \sin A \sin B \sin C = 1$, then the value of $\frac{\sin^2 A}{\sin^2 B} + 2 \frac{\sin^2 B}{\sin^2 C} + \frac{\sin^2 C}{\sin^2 A}$, is equal to (S) 8

14. Column-I Column-II

(A) If the value of $(\tan 18^\circ) (\sin 36^\circ) (\cos 54^\circ) (\tan 72^\circ) (\tan 108^\circ)$ $\times (\cos 126^\circ) (\sin 144^\circ) (\tan 162^\circ) (\cos 180^\circ)$ is $k \sin^2 18^\circ$, then 'k' has the value equal to (Q) $\frac{3}{4}$
(B) If $\sin^3 x \cos 3x + \cos^3 x \sin 3x = \frac{3}{8}$, then the value of $\sin 4x$ is (R) $\frac{5}{4}$
(C) For all permissible values of x, the maximum value of the $f(x) = \frac{5\sin^3 x \cos x}{\tan^2 x + 1}$, is (S) $\frac{5}{8}$

Subjective :

15. If the solution set of the inequality $\log_x \left( \frac{5}{2} \cdot \frac{1}{x} \right) > 1$ is $(a, b) \cup (c, d)$ then the value of $\frac{cd}{ab}$ where $(a < b < c < d)$.

16. The ratios of the lengths of the sides $BC$ and $AC$ of $\triangle ABC$ to the radius of circumscribed circle are equal to 2 and $\frac{3}{2}$ respectively. If the ratio of the lengths of the bisectors of the interior angles $B$ and $C$ is $\alpha(\sqrt{\alpha - 1}) \beta\sqrt{\gamma}$ where $\alpha, \beta, \gamma \in N$. Then find the value of $(\alpha + \beta + \gamma)$.

17. Tangents parallel to the three sides of $\triangle ABC$ are drawn to its incircle. If $x, y, z$ be the lengths of the parts of the tangents within the triangle (with respect to the sides $a, b, c$) then find the value of $\frac{x}{a} + \frac{y}{b} + \frac{z}{c}$. 
Single Choice Question:

1. Two circles of radii \( r_1 \) and \( r_2 \) are both touching the coordinates axes and intersecting each other orthogonally. The value of \( \frac{r_1}{r_2} \) (where \( r_1 > r_2 \)) equals
   (A) \( 2 + \sqrt{3} \)  
   (B) \( \sqrt{3} + 1 \)  
   (C) \( 2 - \sqrt{3} \)  
   (D) \( 2 + \sqrt{5} \)

2. Consider the family of lines \( (x - y - 6) + \lambda (2x + y + 3) = 0 \) and \( (x + 2y - 4) + \mu (3x - 2y - 4) = 0 \). If the lines of these 2 families are at right to each other then the locus of their point of intersection is
   (A) \( x^2 + y^2 - 2x + 4y - 3 = 0 \)  
   (B) \( x^2 + y^2 + 4x + 3y - 3 = 0 \)  
   (C) \( x^2 + y^2 - 3x + 4y - 3 = 0 \)  
   (D) \( x^2 + y^2 - 4x + 3y + 3 = 0 \)

3. The feet of the perpendicular from the origin on a variable chord of the circle \( x^2 + y^2 - 2x - 2y = 0 \) is \( N \). If the variable chord makes an angle of 90º at the origin, then the locus of \( N \) has the equation
   (A) \( x^2 + y^2 - x - y = 0 \)  
   (B) \( x^2 + y^2 + x + y = 0 \)  
   (C) \( x^2 + y^2 - 2x - 2y = 0 \)  
   (D) \( x^2 + y^2 - 2x + 2y = 0 \)

**Paragraph for question nos. 4 to 6**

Consider \( \triangle XYZ \) whose sides \( x, y \) and \( z \) opposite to angular points \( X, Y \) and \( Z \) are in geometric progression.

4. If \( r \) be the common ratio of G.P. then
   (A) \( \frac{\sqrt{5} - 1}{2} < r < \frac{\sqrt{5} + 1}{2} \)  
   (B) \( \frac{\sqrt{5} - 2}{2} < r < \frac{\sqrt{5} + 2}{2} \)  
   (C) \( \frac{\sqrt{5} - 1}{3} < r < \frac{\sqrt{5} + 1}{3} \)  
   (D) \( \frac{\sqrt{5} - 2}{3} < r < \frac{\sqrt{5} + 2}{3} \)

5. The integral values of \( \frac{\sin Y}{\sin X} \) is
   (A) prime only  
   (B) even  
   (C) composite  
   (D) odd

6. The maximum value of \( \frac{\sin Z}{\sin Y} \) is
   (A) irrational number  
   (B) rational number but not integer  
   (C) integer  
   (D) not defined

**Paragraph for question nos. 4 to 6**

Consider two circles \( S_1 : x^2 + y^2 - 9 = 0 \) and \( S_2 : x^2 + y^2 - 10x + 9 = 0 \)

7. Length of the common chord of \( S_1 = 0 \) and \( S_2 = 0 \) is
   (A) \( \frac{6}{5} \)  
   (B) \( \frac{24}{25} \)  
   (C) \( \frac{12}{5} \)  
   (D) \( \frac{24}{5} \)

8. If \( \theta \) is the angle between the two common tangents of \( S_1 = 0 \) and \( S_2 = 0 \), then \( \cos \theta \) equals
   (A) \( \frac{23}{25} \)  
   (B) \( \frac{24}{25} \)  
   (C) \( \frac{12}{25} \)  
   (D) \( \frac{13}{25} \)

9. Distance of a common tangent of \( S_1 = 0 \) and \( S_2 = 0 \) from point \( (9, 0) \) is
   (A) \( \frac{26}{5} \)  
   (B) \( \frac{24}{5} \)  
   (C) \( 5 \)  
   (D) \( \frac{25}{6} \)
Assertion & Reason:

10. **Statement-1**: There lies exactly 3 points on the curve \(8x^3 + y^3 + 6xy = 1\), which form an equilateral triangle

**because**

**Statement-2**: The locus of all point \(P(x, y)\) satisfying \(8x^3 + y^3 + 6xy = 1\) consists of union of a straight line and a point not on the line.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

11. Let \(ABC\) be a triangle with centroid \(G\) and incentre \(I\).

**Statement-1**: If \(GI\) is parallel to the side \(CA\), then \(a\), \(b\), \(c\) are in A.P.

**because**

**Statement-2**: In a triangle, incentre from the angular point A divides the angle bisector in the ratio of \(a:(b + c)\) reckoning from the vertex.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true.

One or more than one correct:

12. In \(\Delta ABC\), \(D\) is a point on \(BC\) such that \(DB = 14\), \(DA = 13\) and \(DC = 4\). If the circumcentre of the \(\Delta ADB\) is congruent to the circumcircle of the \(\Delta ADC\) then which of the following is/are correct?

(A) Angle \(B > 45^\circ\) but angle \(C < 45^\circ\)
(B) both the angles \(B\) and \(C\) are greater than \(45^\circ\)
(C) area of the triangle is 108 sq. units
(D) measure of angle \(A\) equal to \(\tan^{-1}\left(\frac{24}{7}\right)\)

13. Let \(f_n(\theta) = \sum_{n=0}^{\infty} \frac{1}{4^n} \sin^4 (2^n \theta)\). Then which of the following alternative(s) is/are correct?

(A) \(f_2\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\)
(B) \(f_3\left(\frac{\pi}{8}\right) = \frac{2 + \sqrt{2}}{4}\)
(C) \(f_4\left(\frac{3\pi}{2}\right) = 1\)
(D) \(f_5(\pi) = 0\)

14. If one vertex of a equilateral triangle of side 2 lies at the origin and other lies on the line \(x - \sqrt{3}y = 0\), then the coordinates of the third vertex are

(A) \((0, 2)\)
(B) \((-\sqrt{2}, \sqrt{2})\)
(C) \((2, 0)\)
(D) \((-\sqrt{3}, 1)\)
Match the column :

15. Column-I Column-II
(A) If a, b and c are distinct real numbers, such that the quadratic expressions

\[ Q_1(x) = ax^2 + bx + c, \ Q_2(x) = bx^2 + cx + a \] and \[ Q_3(x) = cx^2 + ax + b \]
are always non negative, then the possible integer in the range of the expression \( y = \frac{a^2 + b^2 + c^2}{ab + bc + ca} \), is

(P) 1

(Q) 2

(B) The sum
\[
\frac{2^1}{4^1 - 1} + \frac{2^2}{4^2 - 1} + \frac{2^4}{4^4 - 1} + \frac{2^8}{4^8 - 1} \ldots \infty
\]
is equal to

(R) 3

(C) If \( S_n = \sum_{n=1}^{\infty} \frac{2n+1}{n^3 + 2n^3 + n^2} \), then \( S_{10} \) is less than

(S) 4

(D) The shortest distance from the point \( M(-7, 2) \)
to the circle \( x^2 + y^2 - 10x - 14y - 151 = 0 \), is

(T) 5

Subjective :

16. In an isosceles \( \triangle ABC \), if the altitudes intersect on the inscribed circle then find the secant of the vertical angle ‘A’.

17. Real number \( x, y \) satisfies \( x^2 + y^2 = 1 \). If the maximum and minimum value of the expression \( z = \frac{4 - y}{7 - x} \) are \( M \) and \( m \) respectively, then find the value \( (2M + 6m) \).

18. A cricket player played \( n \) \( (n > 1) \) matches during his career and made a total of \( \frac{(n^2 - 12n + 39)(4.6^n - 5.3^n + 1)}{5} \) runs. If \( T_r \) represent the runs made by the player in \( r^{th} \) match such that

\( T_1 = 6 \) and \( T_r = 3T_{r-1} + 6 \), \( 2 \leq r \leq n \) then find \( n \).
Single Choice Question:

1. Number of distinct point(s) with integer coordinates (both x, y integer) which can lie on a circle with centre \((\sqrt{2}, \sqrt{3})\), is
   (A) 0   (B) 1   (C) 2   (D) more than 2

2. Consider line \(L_1 : y - 3 = 0\) and circle \(S : x^2 + y^2 - 4y + 3 = 0\). The area enclosed by \(S = 0\), \(L_1 = 0\) and the lines which touch the circle and passing through origin, is
   (A) \(\frac{3\sqrt{3}}{2} - \frac{\pi}{2}\)   (B) \(3\sqrt{3} - \frac{\pi}{2}\)   (C) \(3\sqrt{3} - \pi\)   (D) \(3\sqrt{2} + \pi\)

3. Let \(L = 0\) be in a line in parametric form given by \(x = x_1 + r \cos \theta\) and \(y = y_1 + r \sin \theta\) where \(r\) and \(\theta\) have the usual meaning. The line \(L = 0\) has been rotated through \(\alpha\) and \(\frac{\pi}{2} - \alpha\) in clockwise and anticlockwise both directions about points on the line situated at a distance 2 units from the fixed point \(P(x_1, y_1)\). If \(\alpha = 45^\circ\), then area of the figure so formed is
   (A) 4 sq. units   (B) 8 sq. units   (C) \(4\sqrt{2}\) sq. units   (D) No closed figure is formed

4. Consider a family of circles which are passing through \(M(1, 1)\) and are tangent to x-axis. If \((h, k)\) is the centre of circle then
   (A) \(k \geq \frac{1}{2}\)   (B) \(-\frac{1}{2} \leq k \leq \frac{1}{2}\)   (C) \(k \leq \frac{1}{2}\)   (D) \(0 < k < \frac{1}{2}\)

5. Number of integral values of ‘\(k\)’ for which the chord of the circle \(x^2 + y^2 = 125\) passing through \(P(8, k)\) gets bisected at \(P(8, k)\) and has integral slope is
   (A) 8   (B) 6   (C) 4   (D) 2

6. A variable line \(L = 0\) is drawn through \(O(0, 0)\) to meet the lines \(L_1 : x + 2y - 3 = 0\) and \(L_2 : x + 2y + 4 = 0\) at points \(M\) and \(N\) respectively. A point \(P\) is taken on \(L = 0\) such that \(\frac{1}{OP^2} = \frac{1}{OM^2} + \frac{1}{ON^2}\). Locus of \(P\) is
   (A) \(x^2 + 4y^2 = \frac{144}{25}\)   (B) \((x + 2y)^2 = \frac{144}{25}\)   (C) \(4x^2 + y^2 = \frac{144}{25}\)   (D) \((x - 2y)^2 = \frac{144}{25}\)

Paragraph for question nos. 7 to 9:

Consider a circle \(S = 0\) of radius 1 unit and it touches the X-axis at point A. The centre C of this circle lies in the first quadrant. The tangent from \(O\) (where \(O\) is origin) touches the circle at point T. A point \(P\) is taken on this tangent such that \(\Delta OAP\) is right angled at \(A\) and perimeter of \(\Delta OAP\) is 8.

7. The area of \(\Delta OAP\) equals
   (A) \(\frac{5}{3}\)   (B) \(\frac{8}{3}\)   (C) \(\frac{7}{3}\)   (D) \(\frac{4}{3}\)

8. Area of the triangle by the pair of tangents from the origin and the corresponding chord of contact, is
   (A) \(\frac{24}{25}\)   (B) \(\frac{16}{5}\)   (C) \(\frac{8}{5}\)   (D) 2
9. If the circles \( S = 0 \) and \( x^2 + y^2 - 2k^2y + 14 = 0 \) intersect orthogonally, then \( k \) is
\( (A) - 3 \) or \( 2 \) \hspace{0.5cm} (BN) \( -2 \) or \( 2 \) \hspace{0.5cm} (C) \( -2 \) or \( 3 \) \hspace{0.5cm} (D) \( -3 \) or \( 3 \)

**Paragraph for question nos. 10 to 12:**

A circle \( S = 0 \) with centre \( C \) touches the line \( y = x \) at a point \( P \) such that \( OP = 4\sqrt{2} \) (where \( O \) is origin). The circle \( S = 0 \) contains the point \( N(-10, 2) \) in its interior and length of its chord on the line \( x + y = 0 \) is \( 6\sqrt{2} \).

10. Radius of the circle \( S = 0 \), is
\( (A) \ 4\sqrt{2} \) \hspace{0.5cm} (B) \( 5\sqrt{2} \) \hspace{0.5cm} (C) \( 6\sqrt{2} \) \hspace{0.5cm} (D) \( 7\sqrt{2} \)

11. Coordinates of the centre of the circle \( S = 0 \), is
\( (A) (-9, 1) \) \hspace{0.5cm} (B) \( (-7, 2) \) \hspace{0.5cm} (C) \( (-6, 3) \) \hspace{0.5cm} (D) \( (-8, 2) \)

12. Equation of the circle touching the line \( y = x \) at \( P \) and passing through \((-1, 1)\), is
\( (A) \ x^2 + y^2 + 25x - 9y + 32 = 0 \) \hspace{0.5cm} (B) \( x^2 + y^2 + 25x + 9y + 14 = 0 \)
\( (C) \ x^2 + y^2 + 13x - 13y + 24 = 0 \) \hspace{0.5cm} (D) \( x^2 + y^2 + 25x + 9y + 64 = 0 \)

**Paragraph for question nos. 13 to 15:**

Consider two points \( A \equiv (1, 2) \) and \( B \equiv (3, -1) \). Let \( M \) be a point on the straight line \( L \equiv x + y = 0 \).

13. If \( M \) be a point on the line \( L = 0 \) such that \( AM + BM \) is minimum, then the reflection of \( M \) in the line \( x=y \) is
\( (A) \ (1, -1) \) \hspace{0.5cm} (B) \( (-1, 1) \) \hspace{0.5cm} (C) \( (2, -2) \) \hspace{0.5cm} (D) \( (-2, 2) \)

14. If \( M \) be a point on the line \( L = 0 \) such that \( |AM - BM| \) is maximum, then the distance of \( M \) from \( N \equiv (1,1) \) is
\( (A) \ 5\sqrt{2} \) \hspace{0.5cm} (B) \( 7 \) \hspace{0.5cm} (C) \( 3\sqrt{5} \) \hspace{0.5cm} (D) \( 10 \)

15. If \( M \) be a point on the line \( L = 0 \) such that \( |AM - BM| \) is minimum, then the area of \( \triangle AMB \) equals
\( (A) \ \frac{13}{4} \) \hspace{0.5cm} (B) \( \frac{13}{2} \) \hspace{0.5cm} (C) \( \frac{13}{6} \) \hspace{0.5cm} (D) \( \frac{13}{8} \)

**One or more than one correct:**

16. Let \( P \) be a point on the circle \( S \equiv x^2 + y^2 - 6x - 8y + 16 = 0 \). \( OX \) is the positive side of \( X \)-axis where \( 'O' \) is the origin. Which of the following is/are correct?
\( (A) \ \text{OP is minimum when } P \text{ is } \left( \frac{6}{5}, \frac{8}{5} \right) \) \hspace{0.5cm} (B) \( \text{OP is maximum when } P \text{ is } \left( \frac{24}{5}, \frac{32}{5} \right) \)
\( (C) \ \angle POX \text{ is minimum when } P \text{ is } \left( \frac{96}{25}, \frac{28}{25} \right) \) \hspace{0.5cm} (D) \( \angle POX \text{ is maximum when } P \text{ is } (0, 4) \)

17. Which of the following statement(s) holds good?
\( (A) \ \text{Locus of the centre of a variable circle } S = 0 \text{ which cuts two given circles } S_1 = 0 \text{ and } S_2 = 0 \)
\( \text{orthogonally (with coefficient of } x^2 \text{ and } y^2 \text{ unity in } S = 0, S_1 = 0 \text{ and } S_2 = 0) \text{ is } S_1 - S_2 = 0 \)
\( (B) \ \text{The acute angle bisector between the lines } 2x - y + 1 = 0 \text{ and } x - 2y + 2 = 0 \text{ is } 3x - 3y - 1 = 0 \)
\( (C) \ \text{If the lines } px + qy + r = 0, qx + ry + p = 0 \text{ and } rx + py + q = 0 \text{ and concurrent} \)
\( \text{then } \frac{p^2}{qr} + \frac{q^2}{pr} + \frac{r^2}{pq} = 3. \)
\( (D) \ \text{If } \theta \text{ is the acute angle between the line pair } 2x^2 - 5xy + 2y^2 - 6x + 9y + 4 = 0, \text{ then } \cos \theta = \frac{3}{5} \)
18. Tangents PA and PB are drawn to the circles \( S \equiv x^2 + y^2 - 2y - 3 = 0 \) from the point P(3, 4). Which of the following alternative(s) is/are correct?
(A) The power of point P(3, 4) with respect to circle \( S = 0 \) is 14
(B) The angle between tangents from P(3, 4) to the circle \( S = 0 \) is \( \frac{\pi}{3} \)
(C) The equation of circumcircle of \( \Delta PAB \) is \( x^2 + y^2 - 3x - 5y + 4 = 0 \)
(D) The area of quadrilateral PACB is \( 3\sqrt{7} \) square units where C is the centre of circle \( S = 0 \)

19. If the equation of circle touching the y-axis at (0, 3) and making an intercept of 8 unit on x-axis is \( x^2 + y^2 + 2gx + 2fy + c = 0 \), then \( (g + f + c) \) can be
(A) 1 (B) 7 (C) 11 (D) 14

20. Which of the following statements is/are incorrect?
(A) Two circles always have a unique common normal
(B) Radical axis is always perpendicular bisector to the line joining the centres of two circles
(C) Radical axis is nearer to the centre of circle of smaller radius
(D) Two circles always have a radical axis

Match the Column:

21. Column-I | Column-I
---|---
(A) If the equation of the image of line pair, \( y = |x - 2| \) in y-axis is \( y^2 - x^2 - 4x + 3 = \lambda \), then \( \lambda \) equals | (P) 21
(B) Area of the parallelogram formed by the straight lines \( 3x + 4y = 14, 3x + 4y = 7 \); \( 4x + 3y = 35 \) and \( 4x + 3y = 21 \), is | (Q) 2
(C) The radius of the circle whose two normals are represented by the equation \( x^2 - 5xy - 5x + 25y = 0 \) and which touches externally the circle \( x^2 + y^2 - 2x + 4y - 4 = 0 \) will be | (R) 3
(D) Let \( 3y^2 - 8xy + 5x^2 = 0 \) are two tangents from origin to a unit circle in first quadrant. If the length of tangent on this circle from origin is \( a + \sqrt{b} \), then \( (a + b) \) equals | (S) 14

Subjective:

22. The equation of a line through the mid point of the sides AB and AD of rhombus ABCD, whose one diagonal is \( 3x - 4y + 5 = 0 \) and one vertex is A(3, 1) is \( ax + by + c = 0 \). Find the absolute value of \( (a + b + c) \) where a, b, c are integers expressed in lowest form.

23. If the straight line joining the origin to the points of intersection of \( 3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0 \) and \( 2x + 3y = k \) are at right angles, then find the value of \( 5k - 6k^2 \).

24. Let \( S_1 = 0 \) and \( S_2 = 0 \) be two circles intersecting at P(6, 4) and both are tangent to x-axis and line \( y = mx \). If product of radii of the circles \( S_1 = 0 \) and \( S_2 = 0 \) is \( \frac{52}{3} \), then find the value of \( m^2 \).
PRACTICE TEST- 6

Single Choice Question:

1. If \((-2, 6)\) is the image of the point \((4, 2)\) with respect to the line \(L = 0\), then \(L\) is equal to
   \(\text{(A)}\) \(3x - 2y + 11 = 0\) \(\text{(B)}\) \(2x - 3y + 11 = 0\) \(\text{(C)}\) \(3x - 2y + 5 = 0\) \(\text{(D)}\) \(6x - 4y + 1 = 0\)

2. Solution set of the equation
   \[\sqrt{2} (2 \cos 2x - 1) + \sqrt{3 - 4 \cos 2x} \cos 4x = \sqrt{2}\] is
   \(\text{(A)}\) \(n \pi \pm \frac{\pi}{2}; n \in \mathbb{I}\) \(\text{(B)}\) \(2n \pi; n \in \mathbb{I}\) \(\text{(C)}\) \(n \pi; n \in \mathbb{I}\) \(\text{(D)}\) \((2n + 1) \frac{\pi}{2}; n \in \mathbb{I}\)

3. What is the co-efficient of \(x^{-1003}\) in the binomial expansion of \(\left(x - \frac{1}{\sqrt{x}}\right)^{2009}\)
   \(\text{(A)}\) \(-2009\) \(\text{(B)}\) \(2009\) \(\text{(C)}\) \(2010\) \(\text{(D)}\) \(2008\)

4. The number of three digit numbers satisfying the condition that there is no repetition in digits, the number must contain 5, and is less than 800
   \(\text{(A)}\) 170 \(\text{(B)}\) 380 \(\text{(C)}\) 167 \(\text{(D)}\) 168

5. In a certain tournament, a player must be defeated three to be eliminated. If 576 contestants enter the tournament, then the greatest number of games that could be played equals to
   \(\text{(A)}\) 1725 \(\text{(B)}\) 1727 \(\text{(C)}\) 1728 \(\text{(D)}\) 1720

6. In a triangle \(\triangle ABC\) ; line joining incentre and circumcentre is parallel to the side \(AC\). Then the value of \(\cos A + \cos C\) is equal to
   \(\text{(A)}\) \(\frac{1}{2}\) \(\text{(B)}\) \(\frac{1}{\sqrt{2}}\) \(\text{(C)}\) 1 \(\text{(D)}\) \(\frac{\sqrt{3}}{2}\)

7. Co-ordinates of a point \(P\) from which the lengths of tangents drawn to the circle \(x^2 + y^2 + 4x + 7 = 0\), \(2x^2 + 2y^2 + 3x + 5y + 9 = 0\) and \(x^2 + y^2 + y = 0\) are equal, is
   \(\text{(A)}\) \((-1, 3)\) \(\text{(B)}\) \((-1, -2)\) \(\text{(C)}\) \((-2, -1)\) \(\text{(D)}\) None of these

8. If \(x, y \in \mathbb{R}\) and satisfy \((x + 5)^2 + (y - 12)^2 = 14^2\), then the minimum value of \(x^2 + y^2\) is
   \(\text{(A)}\) 2 \(\text{(B)}\) 1 \(\text{(C)}\) \(\sqrt{3}\) \(\text{(D)}\) \(\sqrt{2}\)

Comprehension (Q.9 to Q.11)

Let \(P(a, b)\) be a point in the first quadrant. Circles are drawn through \(P\) touching the co-ordinate axes.

9. Radius of one of the circle be
   \(\text{(A)}\) \((\sqrt{a} - \sqrt{b})^2\) \(\text{(B)}\) \((\sqrt{a} + \sqrt{b})^2\) \(\text{(B)}\) \(a + b - \sqrt{ab}\) \(\text{(D)}\) \(a + b - \sqrt{2ab}\)

10. If two of the circles are orthogonal then \(a\) and \(b\) satisfy the relation
    \(\text{(A)}\) \(a^2 + b^2 = 4ab\) \(\text{(B)}\) \((a + b)^2 = 4ab\) \(\text{(C)}\) \(a^2 + b^2 = ab\) \(\text{(D)}\) \(a^2 - b^2 = 4ab\)

11. Equation of the common chord of the two circles is
    \(\text{(A)}\) \(x + y = a - b\) \(\text{(B)}\) \(x + y = 2\sqrt{ab}\) \(\text{(C)}\) \(x + y = a + b\) \(\text{(D)}\) \(x + y = 4ab\)
Comprehension (Q.12 to Q.14)

In a \( \triangle ABC \) ; If O is circum centre and OE is perpendicular bisector of BC. Of and OG are perpendicular bisectors of AC and AB respectively. R and r denote circumradius and inradius respectively of triangle ABC (triangle is neither right angled nor equilateral).

![Diagram of triangle ABC with perpendicular bisectors]

12. Circum radius of \( \triangle BOE \) is
   \[
   (A) \frac{R \sec (A/2)}{2} \quad (B) \frac{R \sec (A/2)}{4} \quad (C) 2R \sec (A/2) \quad (D) \frac{R \sec (A/2)}{8}
   \]

13. Length of BE is equal to
   \[
   (A) 2R \sin (A/2) \quad (B) 2R \cos (A/2) \quad (C) R \sin (A/2) \quad (D) R \cos (A/2)
   \]

14. The ratio of areas of the triangle GEF and ABC is
   \[
   (A) \frac{Rr}{r} \quad (B) \frac{R}{r} \quad (C) \frac{R}{2r} \quad (D) \frac{R}{4r}
   \]

Match the column :

15. If \( a + b + c = 1 \) and \( (a, b, c > 0) \), then
   \[
   \begin{array}{cc}
   \text{Column-I} & \text{Column-II} \\
   \text{(A)} & \text{abc} \quad (P) \quad \geq 64 \\
   \text{(B)} & \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \quad (Q) \quad \leq \frac{8}{27} \\
   \text{(C)} & \left(1 + \frac{1}{a}\right) \left(1 + \frac{1}{b}\right) \left(1 + \frac{1}{c}\right) \quad (R) \quad \geq 9 \\
   \text{(D)} & (1 - a) (1 - b) (1 - c) \quad (S) \quad \leq \frac{1}{27}
   \end{array}
   \]

16. \[
   \begin{array}{cc}
   \text{Column-I} & \text{Column-II} \\
   \text{(A)} & \text{Number of subsets of \{a, b, c, d, e, f, g\} which contain both a and b, are} \quad (P) \quad 120 \\
   \text{(B)} & \text{Number of different arrangements of the letters in CONTEST in which first two places are occupied by vowels, are} \quad (Q) \quad 32 \\
   \text{(C)} & \text{Number of permutation of the letter ABCDEFG which contain the word BAD are} \quad (R) \quad 465 \\
   \text{(D)} & \text{Number of different ways in which atleast 3 fruits can be selected from 3 apples, 4 mangoes, 5 oranges and 3 bananas is} \quad (S) \quad 120 \\
   & \text{(fruits of the same species are identical)}
   \end{array}
   \]
Single Choice Question:

1. The coefficient of \( a^3 b^4 c^7 \) in the expansion of \((bc + ca + ab)^8\) is
   - (A) 60
   - (B) 120
   - (C) 30
   - (D) 280

2. Three circles of same radius 5 intersect at a point O and each two intersect at A, B, and C respectively. Then the radius of the circle that circumscribes \( \triangle ABC \) is
   - (A) 10
   - (B) 2.5
   - (C) 5
   - (D) 7.5

3. Number of ways in which the letters of the word DECISIONS be arranged so that letter “N” is somewhere to the right of letter “D”, is
   - (A) 8!
   - (B) \( \frac{9!}{4} \)
   - (C) \( \frac{9!}{8} \)
   - (C) 2.89

4. There are 10 stations enroute. A train has to be stopped at 3 of them. Number of ways in which the train can be stopped if at least two of the stopping stations are consecutive, is
   - (A) 54
   - (B) 56
   - (C) 63
   - (D) 64

One or more than one correct:

5. Given that \( \alpha, \gamma \) are the roots of the equation \( Ax^2 - 4x + 1 = 0 \) and \( \beta, \delta \) the roots of the equation \( Bx^2 - 6x + 1 = 0 \). If \( \alpha, \beta, \gamma \) and \( \delta \) are in H.P. then
   - (A) \( A = 3 \)
   - (B) \( B = 6 \)
   - (C) \( A = 4 \)
   - (D) \( B = 8 \)

6. A triangle has altitude of length 5 and 7. The length of third altitude CAN NOT be
   - (A) 17
   - (B) 18
   - (C) 19
   - (D) 20

7. The system of equations \(|x| + |y| = 1, x^2 + y^2 = a^2\) will have
   - (A) four solutions if \( a = 1 \)
   - (B) two solutions if \( a = 1/2 \)
   - (C) four solutions if \( \frac{1}{\sqrt{2}} < a < 1 \)
   - (D) All of the above

8. \((a x + a^2 y + 1)^{2009}\) is a polynomial in \( x \) and \( y \). If the sum of the co-efficients vanishes for some real value of \( a \). Then possible of \( a \) is/are
   - (A) -2
   - (B) -3
   - (C) 1
   - (D) 2

Comprehension (Q.9 to Q.11)

Consider two externally touching circles \( S_1 \) and \( S_2 \) having centres at points A and B whose radii are 1 & 2 respectively. A tangent to circle \( S_1 \) from point B touch the circle \( S_1 \) at point C. D is chosen on circle \( S_2 \) so that AC is parallel to BD and the two segments BC & AD do not intersect. Segment AD intersects the circle \( S_1 \) at E. The line through B and E intersects the circle \( S_1 \) at another point F.

9. The length of segment EF is
   - (A) 2
   - (B) 3
   - (C) \( \frac{2\sqrt{3}}{3} \)
   - (D) \( \sqrt{3} \)

10. The area of triangle ABD is
    - (A) \( \sqrt{2} \)
    - (B) \( \sqrt{4} \)
    - (C) \( \sqrt{6} \)
    - (D) \( \sqrt{8} \)

11. The length of the segment DE is
    - (A) 1
    - (B) \( \sqrt{3} \)
    - (C) 2
    - (D) 3
Subjective:
12. Circle $S_1$ is centered at $(0, 3)$ with radius 1. Circle $S_2$ is externally tangent to circles $S_1$ and also tangent to $x$-axis. If the locus of the centre of the variable circle $S_2$ can be expressed as $y = 1 + \frac{x^2}{\lambda}$. Find $\lambda$.

13. For $0 \leq \theta < 2\pi$, if the point $(2 \cos \theta, 2 \sin \theta)$ lies in the angle between the lines $y = \pm (x - 2)$ in which origin lies, then $\theta$ lies in the interval of length $k\pi$. Then value of $k$ must be.

14. Let $\triangle ABC$ be isosceles and $AB = AC$. Points $M$ and $N$ are mid-points of $AB$ and $AC$ respectively. Medians $MC$ and $NB$ intersects at right angle. If $\left(\frac{AB}{BC}\right)^2 = \frac{p}{q}$ where $p$ and $q$ are relatively prime. Then find the value of $(p + q)$.

15. Let ‘$S$’ be the sum of all divisors of the least natural number having 12 divisors. If $\lambda$ be the sum of the digits of ‘$S$’. Then find $\lambda/3$.

16. Let $O$ be an octagon with vertices labelled $V_1, V_2, \ldots, V_8$ consecutively. All the diagonals of the octagon are drawn except for diagonals between $V_1$ and $V_5$, $V_2$ and $V_6$, $V_3$ and $V_7$, and $V_4$ and $V_8$. Then number of all triangles, whose vertices are vertices of the octagon, and whose edges are the diagonals which have been drawn, is $\lambda$. Find $(\lambda/4)$.

17. Lattice paths are paths consisting of one-unit steps in the positive horizontal or positive vertical directions. Let distinct lattice paths from the point $(-1, 0)$ to the point $(3, 5)$; if at the most one diagonal step (a vertical unit and a horizontal unit at once) is allowed; are $\lambda$. Let $\mu$ be the sum of all digits of $\lambda$. Then find the value of $\mu/2$. 
ANSWER KEY

PRACTICE TEST-1

PRACTICE TEST-2

PRACTICE TEST-3

PRACTICE TEST-4

PRACTICE TEST-5

PRACTICE TEST-6

PRACTICE TEST-7