VKR Classes

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B.Tech., IIT DELHI
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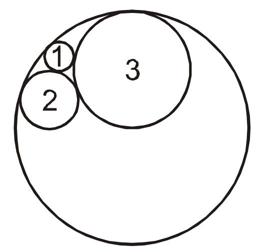
Vitamins For IIT-JEE

School Geometry

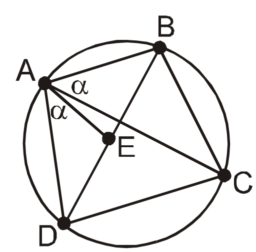
Multiple choice questions with one correct answer **Max. Time : 3 hrs.**

1. On the plane, no more than four circles can be placed so that each circle touches all the others, with every pair touching at a different point. This is illustrated by the picture shown. Given that the three smallest circles have radius of 1, 2 and 3, respectively, what is the radius of the large circle ?



(A) 5 (B) $4\sqrt{3}-1$ (C) 6 (D) $1+3\sqrt{3}$

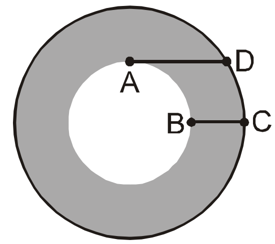
2. Let ABCD be a cyclic quadrilateral (all vertices lie on a circle) with digonals $\overline{AC}$ and $\overline{BD}$ shown in the sketch. Pick point E on the diagonal $\overline{BD}$ so that DAE equals angle BAC. Which of the following statements are true ?



I. Triangle ADE is similar to triangle ACD. II. AE = AB
III. AE . AC = AB . AD IV. $AB^2 + DC^2 = AD^2 + BC^2$

(A) I & II (B) I & III (C) II & IV (D) III & IV

3. In the figure shown, the two circles are concentric and have radii 1 and 2. $\overline{BC}$, if extended, would pass through the centre. $\overline{AD}$ is parallel to $\overline{BC}$ and is tangent to the inner circle. What is the area of the portion ABCD bounded by the two line seqments and the two circular arcs ?

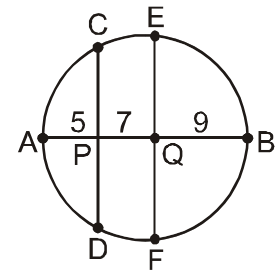


(A) $\frac{3\pi}{8}$ (B) $\frac{\sqrt{5}}{2}+\frac{\pi}{12}$ (C) $\frac{\pi}{2}+\frac{\sqrt{3}}{3}$ (D) $\frac{\sqrt{3}}{2}+\frac{\pi}{12}$

4. A line the sum of whose intercepts is N will be called an N-line. The sum of the y-intercepts of the two 3-lines which pass through the point (–2, –4) is
(A) –3 (B) –1 (C) 1 (D) 3

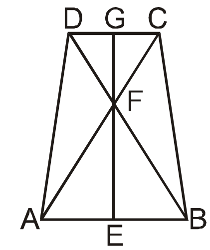
5. In circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at right angles at E. If AE = 20, EB = 36 and CE = 24, what is the circumference of the circle ?
(A) $2\pi\sqrt{117}$ (B) $2\pi\sqrt{793}$ (C) $8\pi\sqrt{34}$ (D) $2\pi\sqrt{61}$

6. In the circle shown, $\overline{AB}$ is the diameter. Chords $\overline{CD}$ and $\overline{EF}$ are perpendicular to AB. The lengths $\overline{AP}, \overline{PQ}$ and $\overline{QB}$ are 5, 7, and 9 respectively. Determine the sum of the lengths of $\overline{CP}$ and $\overline{EQ}$.



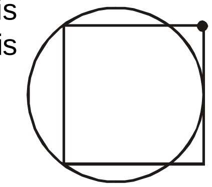
(A) $4\sqrt{5}+2\sqrt{3}$ (B) $2\sqrt{5}+12\sqrt{3}$ (C) $4\sqrt{5}+6\sqrt{3}$ (D) $\sqrt{5}+\sqrt{3}$

7. In the figure shown, the isosceles trapezoid ABCD has base AB = 10 and CD = 6. If the diagonals $\overline{AC}$ and $\overline{DB}$ intersect in point F and the altitude $\overline{GE}$, of length 8 passes through F, then what is the perimeter of the trapezoid ABCD ?



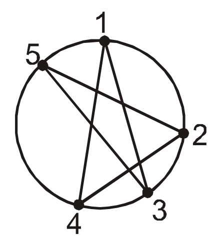
(A) 32 (B) $8\sqrt{17}$ (C) $16 + 4\sqrt{17}$ (D) 45

8. In the figure shown, a circle passes through two adjacent vertices of a square and is tangent to the opposite side of the square. If the side length of the square is 3, what is the area of the circle ?



(A) $\frac{9}{4}\pi$ (B) $\frac{16}{9}\pi$ (C) 6π (D) $\frac{225}{64}\pi$

9. Five points are connected on a circle in the figure shown. What is the sum $m\angle 1 + m\angle 2 + m\angle 4 + m\angle 5$?



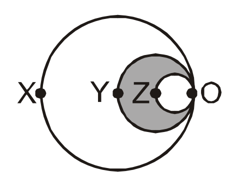
(A) 90° (B) 180° (C) 270° (D) 360°

10. Let A be the ratio of the volume of sphere to the volume of a cube each of whose face is tangent to the sphere, and let B be the ratio of the surface area of this sphere to the surface area of the cube. Then

(A) $A + B = \frac{\pi}{6}$ (B) $A + B = \pi$ (C) $A + B = \frac{2\pi}{3}$ (D) $A + B = \frac{\pi}{3}$

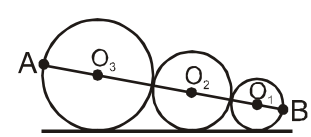
11. The larger of two similar pyramids has 8 times the volume of the smaller. If the smaller pyramid is 5 inches high, how high is the larger pyramid ?
(A) 5 inches (B) 10 inches (C) 20 inches (D) 40 inches

12. In the figure shown, three circles X, Y and Z are tangent to each other at point O. The center of Y is on Z and the center of X is on Y. If the radius of Z is r, what is the area of the unshaded region ?



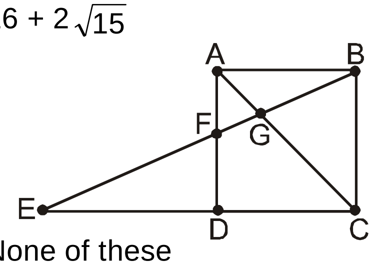
(A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) $13\pi r^2$

13. Three circles are arranged in a row so that each is tangent to the circles next to it. The radii of the two circles at the two ends are 5 and 3. What is the length of the line segment AB that passes through the center of each circle ?



(A) 24 (B) $16 + 4\sqrt{3}$ (C) 20 (D) $16 + 2\sqrt{15}$

14. In the square ABCD, a line through B intersects the extension of $\overline{CD}$ at E, the side $\overline{AD}$ at F and the diagonal $\overline{AC}$ at G. If BG = 9 and GF = 3, then what is the length of EF ?

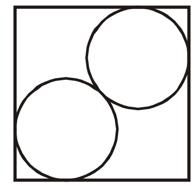


(A) 12 (B) 24 (C) 18 (D) None of these

15. Given two similar triangles, the are of the larger triangle is sixteen times the area of the smaller triangle. Find the ratio of the perimeter of the larger triangle to the perimeter of the smaller triangle

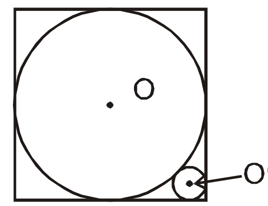
(A) $\sqrt{8}$: 1 (B) 4 : 1 (C) 16 : 1 (D) 32 : 1

16. Two circles each with radius of 1 are inscribed so that their centers lie along the diagonal of the square shown. Each circle is tangent to two sides of the square and they are tangent to each other. Find the area between the circles and the square.



(A) $6 : 4\sqrt{2} - 2\pi$ (B) $10\sqrt{2} - 2\pi$ (C) $10 - 2\pi$ (D) $4 + 2\sqrt{2} - 2\pi$

17. Let circle O be inscribed in a square with side length 1. A smaler circle O′ is inscribed in the lower right corner of the square so that O′ is tangent to O and the two sides of the square. Find the area of the smaller circle.



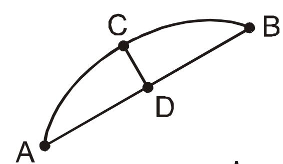
(A) $(41 - 24\sqrt{2})\pi$ (B) $\left(\frac{17+12\sqrt{2}}{4}\right)\pi$

(C) $(17 + 12\sqrt{2})\pi$ (D) $\left(\frac{17-12\sqrt{2}}{4}\right)\pi$

18. A dog is tied to the corner of a house with a regular hexagon base that measures 6 ft on each side If the rope is 12 ft in length, what is the area in square feet of the region outside the house that the dog can reach ?

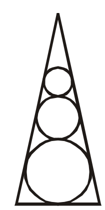
(A) 108π (B) 144π (C) 180π (D) 216π

19. In the figure shown AB is a minor arc of a circle and $\overline{CD}$ is the perpendicular bisecot of chord $\overline{AB}$. If AB = 40 and CD = 8. Find the circumference of the circle.



(A) 8π (B) 24π (C) 40π (D) 58π

20. In the figure shown, three circles are inscribed in a cone as shown. The radius of the circles are 8, 12, and r. Find the area of largest circle with radius r.

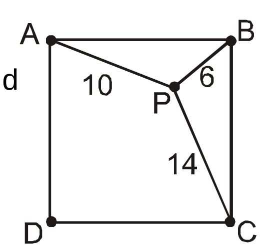


(A) 324π (B) 225π (C) 196π (D) 289π

21. The frustum of a cone has a smaller base with a radius of 6 and a larger base wit a radius of 10. The length of the lateral segment between the base is 12. Determine the volume of the cone.

(A) $\frac{500\sqrt{2}\pi}{3}$ (B) $\frac{1000\sqrt{2}\pi}{3}$ (C) $\frac{1500\sqrt{2}\pi}{3}$ (D) $\frac{2000\sqrt{2}\pi}{3}$

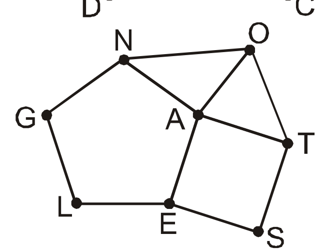
22. In the figure shown, point P is located inside square ABCD. If PA – 10, PB = 6 and PC = 14, find the area of the square.



(A) $8\sqrt{58}$ (B) 140

(C) 232 (D) 464

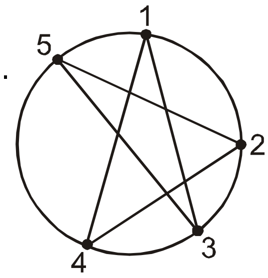
23. In the figure Shown, ANGLE is a regular pentagon, SEAT is a square, and OAT is an equilateral triangle. Determine the measure of $\angle$TON.



(A) 39° (B) 99°

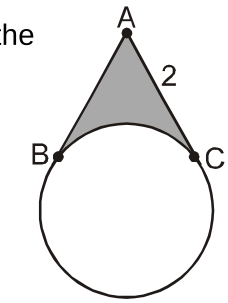
(C) 117° (D) 139°

24. Find, in degrees, the sum of angle 1, 2, 3, 4, 5 in the star-shaped figure shown.



(A) 60°
(B) 90°
(C) 180°
(D) 270°

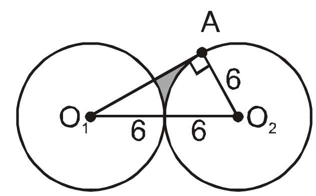
25. In the figure shown, $m\angle A = 60°$, $\overline{AB}$ and $\overline{AC}$ are tangent to the circle. AC = 2. Find the area of the shaded region.



(A) $\frac{3\sqrt{3}-\pi}{2}$ (B) $\frac{4\sqrt{3}}{2}-\pi$

(C) $\frac{4(3\sqrt{3}-\pi)}{9}$ (D) $\frac{2\sqrt{3}-\pi}{3}$

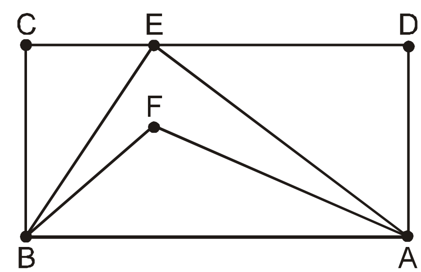
26. In the figure shown, O_1 and O_2 are centers of the circles. $\overline{O_1A}$ is tangent to the circle centered at O_2. Find the area of the shaded region.



(A) $18\sqrt{3} - 7\pi$ (B) $19\sqrt{2} - 8\pi$

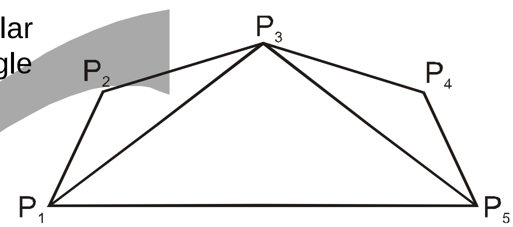
(C) $18\sqrt{3} - 9\pi$ (D) None of these

27. In the figure shown, $m\angle BEA = 100°$. Point F is chosen inside triangle ΔBEA so that line FA bisects $\angle EAB$ and line FB bisects $\angle EBA$. Find the measure of $\angle BFA$.



(A) 140° (B) 145°
(C) 150° (D) 155°

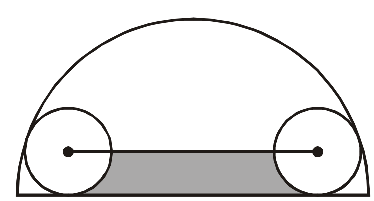
28. In the figure shown, $P_1 P_2 P_3 P_4 P_5$ is a section of a regular dodecagon with each side length of 2. Find the area of triangle $\Delta P_1P_3P_5$.



(A) $3 + 2\sqrt{3}$ (B) $5 + 4\sqrt{3}$

(C) $6 + 2\sqrt{3}$ (D) $8 + 4\sqrt{3}$

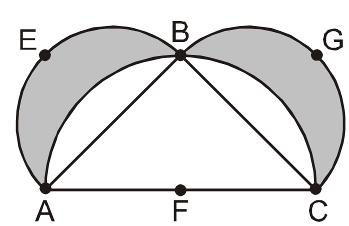
29. In the figure shown, two circles of radius 5 are placed inside a semicircle of radius 18. The two circles are tangent to the diameter and to the semicircle. Find the area of the shaded region ?



(A) $\frac{180-35\pi}{2}$ (B) $\frac{120-25\pi}{2}$

(C) $\frac{240-35\pi}{2}$ (D) $\frac{240-25\pi}{2}$

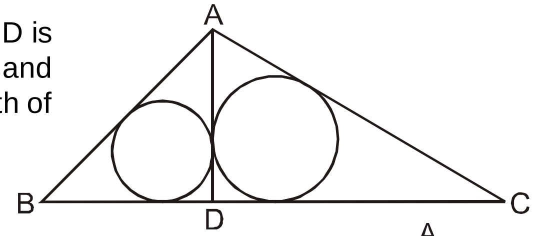
30. In the figure shown, ABC, AEB and CGB are semicircles. F is the midpoint of AC . AF = FC = 1 unit, and AB = BC. What is the area of the shaded region ?



(A) $\frac{1}{2}$ (B) $\frac{\pi}{8}-\frac{1}{2}$

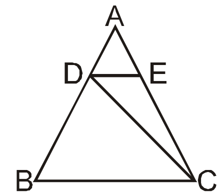
(C) 1 (D) $\frac{\pi}{4}$

31. In triangle ABC, AB = 260, AC = 400 and BD = 520. Point D is chosen on BC so that the circle inscribed in triangle ABD and ADC are tangent to AD at the same point. What is the length of BD ?



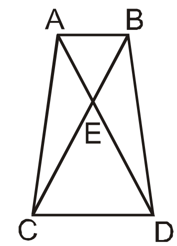
(A) 130 (B) 150
(C) 170 (D) 190

32. In the triangle ABC, $\overline{DE} \parallel \overline{BC}$ and DE : BC = 1 : 4. If the area of ΔADE is 20 square units, then the area of ΔDEC is (in square units) :



(A) 30 (B) 40
(C) 50 (D) 60

33. In the figure shown, $\overline{AB} \parallel \overline{CD}$, AB = 3. If the area of ΔABE is 9 and the area of ΔCDE is 12, then the length of CD is

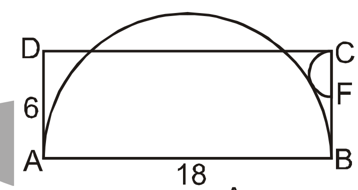


(A) $2\sqrt{3}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $2\sqrt{5}-1$

34. The frusum of a cone has a smaller base with a radius of 3 and a larger base with a radius of 5. The length of the lateral segment between the base is 6. Determine the volume of the cone (in cube units).

(A) $\frac{100\sqrt{2}\pi}{3}$ (B) $150\sqrt{2}\pi$ (C) $\frac{200\sqrt{2}\pi}{3}$ (D) $\frac{250\sqrt{2}\pi}{3}$

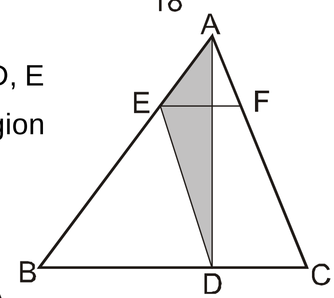
35. In the figure shown, ABCD is a rectangle, AB = 18, AD = 6, and AB is a semicircle with diameter AB, CF is a semicircle with diameter CF. These two arcs are tangent to each other. What is the length of FB ?



(A) $\frac{18}{5}$ (B) $\frac{12}{5}$

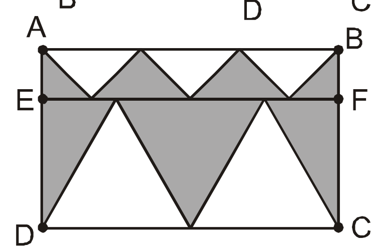
(C) $\frac{21}{5}$ (D) 3

36. In the figure shown, DEFC is a parallelogram with area of 28cm^2. Points D, E and F lie on $\overline{BC}, \overline{AB}$ and $\overline{AC}$, respectively. Find the area of the shaded region (in cm^2).



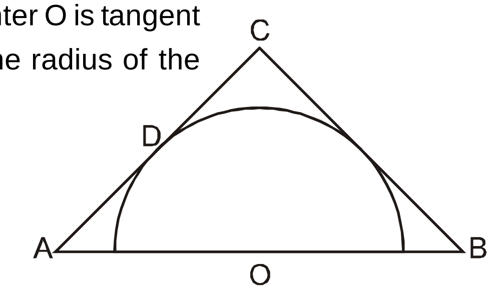
(A) 14 (B) 15
(C) 16 (D) 17

37. In the figure shown, ABCD and CDEF are rectangles, AB = 4 and BC = 3. Find the area ofthe shaded region (in unit square).



(A) 4 (B) 5
(C) 6 (D) 7

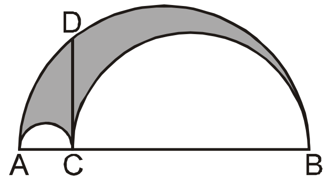
38. In a figure shown, ΔABC is a right triangle. A semicircle with center O is tangent to $\overline{AC}$ and $\overline{BC}$. If the area of ΔABC is S and AB = s, then the radius of the semicircle is



(A) $\frac{S}{\sqrt{s^2+4S}}$ (B) $\frac{2S}{\sqrt{s^2+2S}}$

(C) $\frac{S}{\sqrt{s^2+2S}}$ (D) $\frac{2S}{\sqrt{s^2+4S}}$

39. If the figure shown, AB is a semicircle with deameter $\overline{AB}$. AC is a semicircle with diameter $\overline{AC}$. BC is a semicircle wit diameter $\overline{BC}$. D is a point on AB and $\overline{CD} \perp \overline{AB}$. If CD = 1, what is the area of the shaded region ?

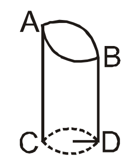


(A) $\frac{1}{3}\pi$ (B) $\frac{1}{4}\pi$ (C) $\frac{1}{5}\pi$ (D) $\frac{1}{6}\pi$

40. A cylindrical tank has a spiral staircase one foot wide attached to its exterior. The staircase goes from the bottom to the top while making exactly 4 complete revolution if the tank is 20 ft high and has a diameter of 16 ft. What is the length of the of the exterior edge of the staircase (in feet) ?

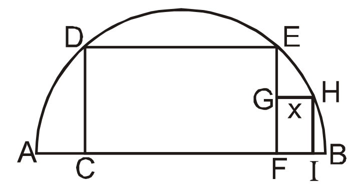
(A) $\sqrt{100+256\pi^2}$ (B) $2\sqrt{100+256\pi^2}$ (C) $\sqrt{100+289\pi^2}$ (D) $4\sqrt{25+324\pi^2}$

41. Given the cylinder as shown, which is cut on a slant. The height goes from 15 in to 21 in, and the radius is 4 in. Find the volume of the cylinder.



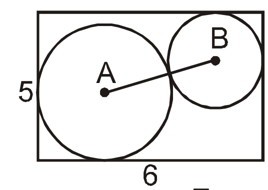
(A) 336π in^3 (B) 288π in^3 (C) 225π in^3 (D) 144π in^3

42. In the figure show, A semicircle has diameter AB. Rectangle CDEF is inscribed in the semicircle with CD = 24 and DE = 56. Square FGHI with side x is between the rectangle and the semicircle as shown. What is the area of FGHI ?



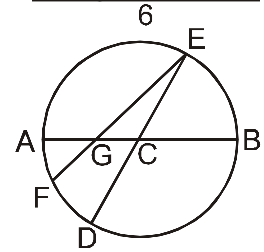
(A) 49 (B) 64
(C) 81 (D) 100

43. In the figure shown, the rectangle has a widt of 6 and height of 5. Circle A has radius r and circle B has radius 1. Find the value of r.



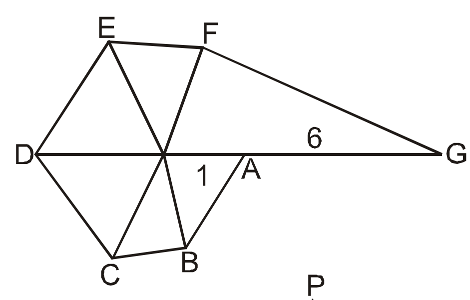
(A) $10-\sqrt{35}$ (B) $\sqrt{35}-5$
(C) $12-\sqrt{35}$ (D) $5+\sqrt{35}$

44. In the figure shown, $\overline{AB}$ and $\overline{ED}$ are diameters of the given circle, intersecting on the center C of the circle. Also, F is the midpoint of the minor arc determined by points A and D, and the chord $\overline{EF}$ ingtersects $\overline{AB}$ on the point G. If $\angle$BCE has measure 60°, then the measure of $\angle$AGF is



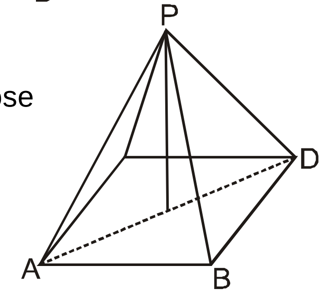
(A) 15° (B) 30° (C) 45° (D) 60°

45. In the figure shown, six similar triangles are each sharing one side with the next triangle and all are sharing one vertex. All angles at that vertex measure 60°. If the side of the last (smallest) triangle that s adjoining the first triangle is 1/6 as large as the longest side of the first triangle, how many time larger is the area of the largest triangle as compared to the smallest ?



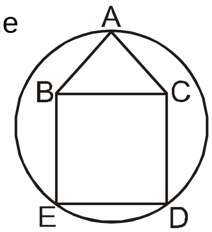
(A) 6 (B) $\sqrt[6]{6^5}$
(C) $\sqrt[3]{6^5}$ (D) 36

46. Find the height of a square pyramid formed by four equilateral triangles whose sides all have length 2.



(A) 1 (B) $\frac{\sqrt{6}}{2}$
(C) $\sqrt{2}$ (D) $\sqrt{3}$

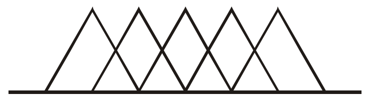
47. In the figure shown, ABEDC is circumscribe a circle through points A, D and E. ABC is an equilateral triangle with side length 2, and BCDE is a square. Find the radius of the circle.



(A) $\frac{\sqrt{3}}{2} + 1$ (B) 2

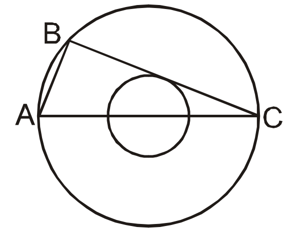
(C) $\sqrt{3} + 1$ (D) $5 - 2\sqrt{3}$

48. Five equilateral triangles, each with side $2\sqrt{3}$, are arranged so they are all on the same side of a line containing one side of each. Along this line, the midpoint of the base of one triangle is the vertex of the next. The area of the region of the plane that is covered by the union of the five triangular regions is



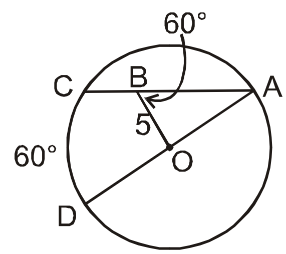
(A) 10 (B) 12 (C) $10\sqrt{3}$ (D) $12\sqrt{3}$

49. The ratio of the radii of two concentric circle is 1 : 3. If $\overline{AC}$ is a diameter of the larger circle, $\overline{BC}$ is a chord of the larger circle that is tangent to the smaller circle, and AB = 12, then the radius of the larger circle is



(A) 13 (B) 18 (C) 21 (D) 24

50. In a circle with center O, AD is the diameter, ABC is a chord, BO = 5 and $\angle ABO = CD = 60°$. Then the length of BC is



(A) 3 (B) $3+\sqrt{3}$ (C) $5 - \frac{\sqrt{3}}{2}$ (D) 5

ANSWERS

1. C	**2.** B	**3.** D	**4.** C	**5.** B	**6.** C	**7.** C
8. D	**9.** B	**10.** D	**11.** B	**12.** D	**13.** D	**14.** B
15. B	**16.** A	**17.** D	**18.** A	**19.** D	**20.** A	**21.** D
22. C	**23.** B	**24.** C	**25.** C	**26.** C	**27.** A	**28.** A
29. D	**30.** C	**31.** D	**32.** D	**33.** A	**34.** D	**35.** A
36. A	**37.** C	**38.** D	**39.** B	**40.** D	**41.** B	**42.** B
43. A	**44.** C	**45.** C	**46.** C	**47.** B	**48.** D	**49.** B
50. D						

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Vitamins For IIT - JEE

Number Theory

Multiple choice questions with one correct answer **Max. Time : 3 hrs.**

1. The last decimal digit of 2004^{2004} is
(A) 2 (B) 4 (C) 6 (D) 8

2. The notation N(x) means the number of prime number less than x. What is the value of N(N(30))? (Remember that 1 is not a prime number.)
(A) 4 (B) 5 (C) 10 (D) 23

3. The number of natural numbers less than 400 that are not divisible by 17 or 23 is
(A) 360 (B) 376 (C) 359 (D) 382

4. The remainder if $1 + 2 + 2^2 + 2^3 + ... + 2^{1999}$ is divided by five is
(A) 0 (B) 1 (C) 2 (D) 3

5. If m and n are positive integers and m + n + mn + 1 = 91 then m + n equals
(A) 15 (B) 17 (C) 18 (D) 19

6. If n! = n × (n – 1) × (n – 2) × ...... × 1, then the number of perfect squares in the infinite sequence
1! . 1! + 2!, 1! + 2! + 3!, ... , 1! + 2! + 3! + 4! + ...+ n!, ... is
(A) 0 (B) 2 (C) 3 (D) monre than 5

7. If a, b and c are integers with 0 < a < b < c and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 1$, the value of c is
(A) 6 (B) 8 (C) 4 (D) impossible to determine

8. Of the numbers below, the largest one that divides every term of the sequence
$2^5 – 2, 3^5 – 3, ..., n^5 – n,.....$, is
(A) 2 (B) 3 (C) 6 (D) 5

9. When $33333^2 + 22222$ is written as a single decimal number, the sum of its digits is
(A) 15 (B) 25 (C) 22 (D) 10

10. If the digits k, m, n of the 3-digit number kmn satisfy 64k + 8m + n = 403, then the number kmn is
(A) 623 (B) 563
(C) 643 (D) not possible to be uniquely determined

11. The number $1 + 3 + 9 + ... + 3^{23}$ is divisible by exactly two numbers between 724 and 734. What are they ?
(A) 728 and 730 (B) 726 and 728 (C) 726 and 730 (D) 730 and 732

12. The sum of a certain number of positive integers is 31. What is the biggest their product can be ?
(A) 55296 (B) 78732 (C) 118098 (D) 49152

13. Evaluate $[\sqrt[3]{1}]+[\sqrt[3]{2}]+......+[\sqrt[3]{124}]$
(A) 401 (B) 402 (C) 403 (D) None of these

14. Let $x_1 < x_2 < x_3 < x_4$ be four numbers. The four numbers can be paired in six different ways, and here are the average values of these pairings : 3, 4, 5, 7, 8 and 9. What is the largest possible value of x_4 ?
(A) 10 (B) 12 (C) 14 (D) 16

15. The largest value of n such that 3^n divides into $1 \times 3 \times 5 \times 7 \times 9 \times .... \times 31$ without remainder is
(A) 5 (B) 6 (C) 7 (D) 8

16. For how many whole numbers between 100 and 999 does the product of the ones digit and tens digit equal the hundreds digit ?
(A) 20 (B) 23 (C) 21 (D) 25

#17. The number of natural numbers that leave a remainder of 41 when divided into 1997 is
(A) 6 (B) 4 (C) 3 (D) 8

18. The number of pairs of integers (m; n) which satisfy the equation $m(m + 1) = 2^n$ is
(A) 1 (B) 2 (C) 3 (D) more than 3

19. If n can be any natural number, how many different values for the remainder can you get if you divide n^2 by 7 ?
(A) 2 (B) 3 (C) 4 (D) 5

20. A book with 12 pages needs the 15 digits 1, 2, 3, 4, 5, 6, 7, 8, 1, 0, 1, 1, 1, 2 in order to number all the pages. Which one of the following numbers cannot be the number of digits needed in order to number all the pages of a book ?
(A) 543 (B) 1998 (C) 1999 (D) 2001

21. The integers from 1 to 2001 are written in order around a circle. Starting at 1, every 6th number is marked (that is 1, 7, 13, 19, etc.). This process is continued until a number is reached that has already been marked. How many unmarked numbers remain ?
(A) None (B) 1668 (C) 1669 (D) 1334

22. Which on of the numbers below can be expressed as the sum of the squares of 6 odd integers ?
(A*) 1998 (B) 1996 (C) 2000 (D) 2004

23. If m and n are positive integers such that $m^2 + 2n = n^2 + 2m + 5$, then the value of n is
(A) 4 (B) 3 (C) 1 (D) impossible to determine

24. If n is a perfect square, then the next perfect square greater than n is
(A) $n^2 + 1$ (B) $n^2 + n$ (C) $2n + 1$ (D) $n + 2\sqrt{n} + 1$

25. Let k be the smallest positive integer with the property that for all n such that $2 \le n \le 10$, when divided by n it leaves a remainder of $n - 1$. Find the sum of the digits of k.
(A) 12 (B) 13 (C) 15 (D) 17

26. Express the sum of the repeating decimals .68686868.... + .07777777777...... as a repeating decimal.
(A) .7656565656... (B) .7666666666.... (C) .767676767676... (D) .76464646......

27. The *n*th triangular number is defined to be the sum of the first n positive integers. For example, the 4th triangular number is 1 + 2 + 3 + 4 = 10. In the first 100 terms of the sequence
1, 3, 6, 10, 15, 21, 28,.....
of trianglar numbers, how many are divisible by 7 ?
(A) 25 (B) 26 (C) 27 (D) None of these

28. The smallest positive integer x for which $1260x = N^3$ where N is an integer, is
(A) 1050 (B) 1260 (C) 7350 (D) 44100

29. How many solutions in positive integers are there for the equation $2x + 3y = 763$?
(A) 255 (B) 254 (C) 128 (D) 127

30. If $\frac{97}{19} = w + \cfrac{1}{x + \cfrac{1}{y}}$, where w, x, y are all integers, then w + x + y equals
(A) 16 (B) 17 (C) 18 (D) 19

31. The unit (ones) digit in the product $(5 + 1)(5^2 + 1)(5^3 + 1).....(5^{2005} + 1)$ is
(A) 6 (B) 5 (C) 2 (D) 1

32. The average of the nine numbers
9 99 999 9999 99999 999999 9999999 99999999 999999999
is a nine-digit number M, all of whose digits are different. The number M does not contain the digit
(A) 7 (B) 1 (C) 5 (D) 0

33. If $3 \times 2^a + 5^b + 7^c + 11^d = 2008$ with a, b, c and d all non-negative integers, then a + b + c + d equals
(A) 6 (B) 7 (C) 8 (D) 9

34. How many positive integers n have the property that bot n and n + 1001 are perfect squares ?
(A) 2 (B) 3 (C) 4 (D) 5

$35. The complex numbers α and β satisfy the relations $\alpha^2 = -1$ and $\beta^2 = -1 - \beta$. How many distinct numbers are formed when we compute all possible products $\alpha^r . \beta^8$ for positive integers r, s ?
(A) 4 (B) 6 (C) 8 (D) 12

36. The set of all real numbers x for which $x + \sqrt{x^2+1} - \frac{1}{x + \sqrt{x^2+1}}$ is a rational number is the set of all
(A) integers x
(B) rational x
(C) real x
(D) x for which $\sqrt{x^2+1}$ is rational

37. If we divide 344 by d the remainder is 3, and if we divide 715 by d the remainder is 2. Which of the following is true about d ?
(A) $10 \le d \le 19$ (B) $20 \le d \le 29$ (C) $30 \le d \le 39$ (D) $40 \le d \le 49$

$38. There are four consecutive integers such that the sum of the cubes of the first three numbers equals the cube of the fourth number. Find the sum of the four numbers.
(A) 12 (B) 16 (C) 18 (D) 22

39. Let N = <u>abcde</u> denote the five ditit number with digits a, b, c, d, e and $a \neq 0$. Let N′ = <u>edcba</u> denote the reverse of N. Suppose that N > N′ and that N – N′ = 5x014 where x is a digit. What is x ?
(A) 4 (B) 6 (C) 7 (D) 8

40. Find an orderd pair (n, m) of positive integers satisfying $\frac{1}{n} - \frac{1}{m} + \frac{1}{mn} = \frac{2}{5}$. What is mn ?
(A) 5 (B) 10 (C) 15 (D) 20

41. The product of three consecutive non-zero integers is 33 times the sum of the three integers. What is the sum of the digits of this products ?
(A) 6 (B) 12 (C) 16 (D) 18

42. It is possible that the difference of two cubes is a perfect square. For example, $28^2 = a^3 - b^3$ for certain positive integers, a and b. In this example, what is a + b?
(A) 12 (B) 14 (C) 16 (D) 18

43. Four positive integers a, b, c and d satisfy abcd = 10!. What is the smallest possible sum a + b + c + d ?
(A) 170 (B) 175 (C) 178 (D) 183

44. If x and y are integers, under what conditions is $x^2 + xy + (x - y)$ odd ?
(A) x is odd and y is odd
(B) x is odd and y is even
(C) x is even and y is even
(D) x is even and y is odd

45. Let A, B and C be digits satisfying $\begin{array}{r} A\ \ B \\ +\ \ A\ \ A \\ \hline C\ \ B\ \ 2 \end{array}$ What is A + B + C ?

(A) 10 (B) 11 (C) 12 (D) 13

$46. A certain integer N has exactly eight factors, counting itself and 1. The numbers 35 and 77 are two of the factors. What is the sum of the digits of N ?

(A) 9 (B) 10 (C) 16 (D) 18

47. Consider the number x defined by the periodic continued fraction $x = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+....}}}}$.

Then x =

(A) $\frac{1}{3}$ (B) $\frac{3}{7}$ (C) $\frac{-3-\sqrt{15}}{2}$ (D) $\frac{-3+\sqrt{15}}{2}$

48. Suppose x and y are integers satisfying both $x^2 + y = 62$ and $y^2 + x = 176$. What is x + y ?

(A) 20 (B) 21 (C) 22 (D) 23

49. What is the smallest positive integer n > 150 such that $\binom{n}{151}$ is divisibly by $\binom{n}{150}$ but not equal to it?

(A) 302 (B) 252 (C) 352 (D) 452

50. Let n denote an integer greater than 1. Which of the following statements are true ?

I. If $2^n - 1$ is prime, then n is prime.

II. If n is prime, then $2^n - 1$ is prime.

III. If $2^n - 1$ is prime, then $2^{n-1} + 1$ is prime.

(A) I (B) I & II (C) II & III (D) III

ANSWERS

1. C	**2.** A	**3.** A	**4.** A	**5.** C	**6.** B	**7.** A
8. C	**9.** D	**10.** A	**11.** A	**12.** B	**13.** D	**14.** B
15. D	**16.** B	**17.** A	**18.** B	**19.** C	**20.** C	**21.** D
22. A	**23.** B	**24.** D	**25.** D	**26.** D	**27.** D	**28.** C
29. D	**30.** A	**31.** A	**32.** D	**33.** D	**34.** C	**35.** D
36. B	**37.** C	**38.** C	**39.** D	**40.** B	**41.** D	**42.** C
43. B	**44.** C	**45.** D	**46.** C	**47.** D	**48.** A	**49.** D
50. A						

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Vitamins For IIT - JEE

Basic Inequalities

Multiple choice questions with one correct answer **Max. Time : 3 hrs.**

1. If $0 < x < y < 1$, which one of the following is not necessarily true ?
(A) $(x – y)^2 < y^2$ (B) $x^2 < 2y^2$ (C) $x^3 – y^3 < x^2$ (D) $(x – y)^2 < x^2$

2. If $x^2 + 6x + y^2 – 4y + 13 = 0$, then $x + y$ is
(A) –1 (B) –2 (C) 1 (D) impossible to determine

3. Suppose that $w = (0.001)^{1000}$, $x = (0.001)^{0.001}$ $y = (1.001)^{1000}$, and $z = (2^{1000} – 1)^{0.001}$. Put these numbers in order from smallest to largest.
(A) w, x, y, z (B) w, x, z, y (C) x, w, y, z (D) x, w, z, y

$4. Let $p(x) = x^2 + x – 2$ and let $q(x) = x^3 + ax^2 + bx + c$. For each real number r, we will assume that $q(r) = 0$ if and only if $p(r) = 0$. What is the largest possible value of $a – b + c$?
(A) 3 (B) 4 (C) 5 (D) 6

5. Consider the following regions in the plane :
$R_1 = \{(x, y) : 0 \le x \le 1 \text{ and } 0 \le y \le 1\}$
$R_2 = \{(x, y) : x^2 + y^2 \le 4/3\}$
The are of the region $R_1 \cap R_2$ can be expressed as $\dfrac{a\sqrt{3} + b\pi}{9}$ where a and b are integers.
Find a + b.
(A) 2 (B) 3 (C) 4 (D) 5

6. A triangle with sides of length 13, 14 and 15 inches is to be cut whole from a rectangular sheet of paper. Expressed in square inches, what is the minimum area that this rectangular sheet can have ?
(A) 168 (B) 174 (C) 188 (D) 202

7. If $\dfrac{23}{30} = \dfrac{1}{a_1} + \dfrac{1}{a_2} + ... + \dfrac{1}{a_n}$, where $a_1, a_2, ..., a_n$ are natural numbers, then the smallest value of n is
(A) 30 (B) 2 (C) 3 (D) 4

8. If x and y are natural numbers and $19x + 97y = 1997$ then the smallest value of $x + y$ is
(A) 21 (B) 23 (C) 38 (D) 41

9. Let n be fixed positive itneger. The maximum value of A such that
$x^n + x^{n-2} + x^{n-1} + ... + \dfrac{1}{x^{n-4}} + \dfrac{1}{x^{n-2}} + \dfrac{1}{x^n} \ge A$ for all positive real numbers x, is
(A) n^2 (B) $n + 1$ (C) 0 (D) $n(n + 1)$

10. The solution set of $1 + x + x^2 + x^3 + ... + x^{1997} \le 0$ is

(A) $\{x : x \le -2\}$ (B) $\{x : x \le -1\}$ (C) $\{x : -1 \le x \le 0\}$ (D) $\{x : x \le 0\}$

11. The solution set of the inequality $|2x - 1| - |x + 3| < 4$ can be written as the union of intervals. The sum of the lengths of these intervals is :

(A) 3.5 (B) 6 (C) 7.5 (D) 10

12. Let S be the solutions set of the inequality $|x^2 - 8x| \ge |x^2 - 8x + 2|$. What is the smallest value of r for which S is also the solution set of the inequality $|x^2 - 8x + c| \le r$ for some real number c ?

(A) 13/2 (B) 15/2 (C) 8 (D) 17/2

13. If A = .7, B = $(1/3)^{(1/3)}$, and C = $(1/2)^{(1/2)}$. Put A, B, and C in increasing order.

(A) BAC (B) ABC (C) BCA (D) CBA

14. Find the area of the planar region difined by $1 \le |x| + |y|$ and $x^2 - 2x + 1 \le 1 - y^2$.

(A) $(3/4)\pi$ (B) $\pi/2$ (C) π (D) $\pi/4$

15. If x is the largest root of $(\log_{10} x)^2 - 4 \log_{10} x = 7$, what is true about x/100 ?

(A) It is between 1000 and 2000 (B) It is between 2000 and 3000

(C) It is between 3000 and 4000 (D) It is bigger than 4000

16. What is the area (in square units) of the region in the first quadrant defined by $18 \le x + y \le 20$?

(A) 36 (B) 38 (C) 40 (D) 42

17. Suppose a, b and c ar real numbers for which $\frac{a}{b} > 1$ and $\frac{a}{c} < -1$. Whcih of the following must be correct ?

(A) $a + b - c > 0$ (B) $a > b$ (C) $(a - c)(b - c) > 0$ (D) $a - b + c > 0$

18. How many ordered pairs of integers with a sum 23 have a product that is maximal ?

(A) 0 (B) 1 (C) 2 (D) 3

19. Suppose {a, b, c, d, e, f} = {2, 3, 4, 5, 6, 7}. What is the least possible value of ab + cd + ef ?

(A) 50 (B) 52 (C) 53 (D) 60

S20. Let the function f be defined by $f(x) = x^2 + 40$. If m is a positive number such that f(2m) = 2f(m) which of the following is true ?

(A) $0 < m \le 4$ (B) $4 < m \le 8$ (C) $8 < m \le 12$ (D) $12 < m \le 16$

21. Find the largest possible integer n such that $1 + 2 + 3 + ... n \le 200$

(A) 14 (B) 17 (C) 19 (D) 21

22. For how many integer values of n is $\frac{3}{17} < \frac{n}{68} < \frac{32}{51}$?

(A) 28 (B) 29 (C) 30 (D) 32

23. How many pairs of positive integer (a, b) with $a + b \le 100$ satisfy $\frac{a + b^{-1}}{a^{-1} + b} = 13$?

(A) 3 (B) 4 (C) 5 (D) 7

24. What is the smallest value of the positive integer n for which

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n.(n+1)}$$ is at least 1 ?

(A) 100 (B) 1000

(C) 2002 (D) there is no such value of n

25. Let $S = 1 + 1/2^2 + 1/3^2 + ... + 1/100^2$. Which of the following is true ?

(A) $S < 1.40$ (B) $1.40 \le S < 2$ (C) $2 \le S < 4$ (D) $4 \le S < 100$

26. If m and n are requried to be integers, how many solutions (m, n) are there to the pair of conditions $5n - 3m = 15$ and $n^2 + m^2 \le 16$?
(A) 0 (B) 1 (C) 2 (D) 3

$27. For $x \ge 0$, what is the smallest possible value of the expression $\log(x^3 - 4x^2 + x + 26) - \log(x + 2)$?
(A) log 3 (B) log 2 (C) log 5 (D) log 4

28. Given p > 0 and m > n, compare the value of m, n and $\frac{m+np}{1+p}$.

(A) $n < m < \frac{m+np}{1+p}$ (B) $\frac{m+np}{1+p}$ (C) $n < \frac{m+np}{1+p} < \frac{m}{2}$ (D) $n < \frac{m+np}{1+p} < m$

29. Suppose a, b, c are nonnegative numbers, and $3a + 2b + c = 5$, $2a + b - 3c = 1$. Find the maximum value of $S = 3a + b - 7c$.

(A) $-\frac{5}{7}$ (B) $-\frac{1}{11}$ (C) $\frac{1}{11}$ (D) $\frac{5}{7}$

$30. Solve the inequality $x^{1+\log_{\frac{1}{2}} x} > \frac{1}{4}x$.

(A) $\frac{1}{2} < x < 2$ (B) $2^{-\sqrt{2}} < x < 2^{\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}} < x < \sqrt{2}$ (D) $\sqrt{2}^{-\sqrt{2}} < x < \sqrt{2}^{\sqrt{2}}$

31. Which ones of the following statements are true ?
(i) $\log_2(n + 3) > \log_2(n + 2)$.
(ii) $\log_2(n + 2) > \log_3(n + 2)$.
(iii) $\log_2(n + 2) > \log_3(n + 3)$.
(A) (ii) (B) (i) and (ii) (C) and (ii) (D) (i), (ii) and (iii)

32. Find the number of integer pairs (x, y) satisfying the inequality : $1 \le x^{2/3} + y^{2/3} \le 2$.
(A) 4 (B) 5 (C) 12 (D) 13

33. Let $f(x) = ax^3 + bx + c$ be any quadratic with real coefficients a, b, c having the property that $|f(x)| \le 1$ for $0 \le x \le 1$. Find the smallest number M such that $|a| + |b| + |c| \le M$.
(A) 1 (B) 17 (C) 3 (D) 38

34. Find the greatest integer n for whcih there exists a simultaneous solutions x to the inequalities
$k < x^k < k + 1$, $k = 1, 2, 3, \ldots, n$.
(A) 2 (B) 4 (C) 6 (D) 8

35. The sides of a triangle are $\sqrt{2}, \sqrt{3}$ and $\sqrt{11}$. Which of the following best describes the triangle ?
(A) Isoscales (B) Nonexistent (C) Acute (D) Equilateral

36. Find the possible values of k so that two lines $kx + y = 3$ and $x - y = 2$ intersect in the first quadrant.

(A) $k > \frac{3}{2}$ (B) $-1 < k \le -\frac{1}{2}$ (C) $k > 1$ (D) $-1 < k < \frac{3}{2}$

37. The first three terms of a geometric sequence are x, y, z and these have a sum of 42. If the middle terms y is multiplied by 5/4, the numbers x, $\frac{5y}{4}$, z now form an arithmetic sequence. What is the largest possible value of x ?
(A) 24 (B) 6 (C) 28 (D) 30

38. If m is the minimum value attained by $f(x, y) = x^2 + y^2 - 10x + 6y + 27$ then
(A) $-15 < m < -12$ (B) $-12 < m < -9$ (C) $-9 < m < -6$ (D) $-6 < m < -3$

39. Let x and y be two positive real numbers satisfying $x + y + xy = 10$ and $x^2 + y^2 = 40$. What integer is nearest $x + y$?
(A) 4 (B) 5 (C) 6 (D) 7

40. If $f(x,y) = (\max(x, y))^{\min(x, y)}$ and $g(x, y) = \max(x, y) - \min(x, y)$, then $f(g(-1, -1\frac{3}{2}), g(-4, -1.75)) =$
(A) –0.5 (B) 0.5 (C) 1 (D) 1.5

41. How many integer n satisfy $|n^3 - 222| < 888$?
(A) 17 (B) 18 (C) 19 (D) 20

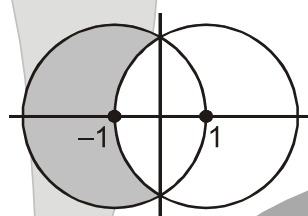
42. How many points (x, y) satisfy the equation $|x^2 - 1| + |y^2 - 4| = 0$?
(A) 2 (B) 4 (C) 6 (D) infinitely many

43. The numbers x, y and z satisfy $|x + 2| + |y + 3| + |z - 5| = 1$. Which of the following could be $|x + y + z|$
(A) 0 (B) 2 (C) 5 (D) 7

44. The set of points satisfying the three inequalities $y \geq 0$, $y \leq x$, and $y \leq 6 - x/2$ is a triangular region with an area of
(A) 12 (B) 18 (C) 24 (D) 36

45. If a, b, c and d are four positive numbers such that $\frac{a}{b} < \frac{c}{d}$, then
(A) $ab < dc$ (B) $a + c < b + d$ (C) $a + d < b + c$ (D) $\frac{a+c}{b+d} < \frac{c}{d}$

46. The shaded region is decribed by



(A) $(x + 1)^2 + y^2 \leq 4$ or $(x - 1)^2 + y^2 \geq 4$
(B) $(x + 1)^2 + y^2 \geq 4$ and $(x - 1)^2 + y^2 \geq 4$
(C) $(x + 1)^2 + y^2 \leq 4$ and $(x - 1)^2 + y^2 \geq 4$
(D) $(x + 1)^2 + y^2 \leq 2$ and $(x - 1)^2 + y^2 \leq 2$

47. Let f be a linear function with the properties that $f(1) \leq f(2)$, $f(3) \geq f(4)$, and $f(5) = 5$. Which of the following statements is true ?
(A) $f(0) < 0$ (B) $f(0) = 0$ (C) $f(1) < f(0) < f(-1)$ (D) $f(0) = 5$

48. The sum of the greatest integer less than or equal to x and the least integer greater than or equal to x is 5. The solution set for x is
(A) $\frac{5}{2}$ (B) $\{x|2 \leq x \leq 3\}$ (C) $\{x|2 \leq x < 3\}$ (D) $\{x|2 < x < 3\}$

49. If a, b and c are real and different and $u = a^2 + 4b^2 + 9c^2 - 6bc - 3ca - 2ab$, then u is always
(A) Non-negative (B) zero (C) Non-positive (D) None of these

50. If a, b, c are positive real numbers, such that $a + b + c = 2$, then
(A) $a^{-1} + b^{-1} + c^{-1} \geq 6$
(B) $(2 - a)(2 - b)(2 - c) \leq 64/27$
(C) $(2 - a)(2 - b)(2 - c) \geq 8abc$
(D) $(1 - a)(1 - b)(1 - c) \geq 6abc$

ANSWERS

1. D	2. A	3. B	4. C	5. C	6. A	7. C
8. B	9. B	10. B	11. D	12. B	13. A	14. A
15. B	16. D	17. C	18. C	19. C	20. B	21. B
22. C	23. D	24. D	25. B	26. B	27. D	28. D
29. B	30. B	31. D	32. C	33. B	34. B	35. B
36. D	37. A	38. C	39. D	40. D	41. C	42. B
43. A	44. C	45.	46. C	47.	48. D	49. A
50. C						

Vitamins For IIT - JEE

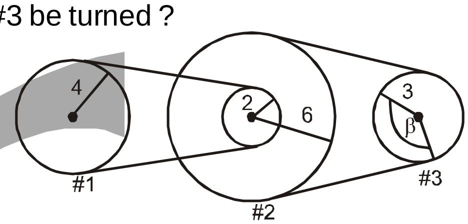
Logical Reasoning

Multiple choice questions with one correct answer **Max. Time : 3 hrs.**

1. In a survey of 69 people, only 9 liked all three of brands A, B, and C; 12 didn't like any of the three; 9 liked only A; 30 disliked A but liked at least one of the other two. If 15 liked exactly two of the three, 12 liked only B, and 31 liked C, how many liked A and B but not C ?

(A) 4 (B) 5 (C) 7 (D) 9

2. In the gear system below the two gears in the centre (#2) turn together. The radius of gear # 1 is 4 cm, the smaller radius in system #2 is 2 cm and the larger is 6 cm, and the radius of #3 is 3 cm. If gear #1 is turned through an angle $\theta = \frac{2\pi}{3}$, through that angle β will gear #3 be turned ?



(A) 2π (B) $\frac{4\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{8\pi}{3}$

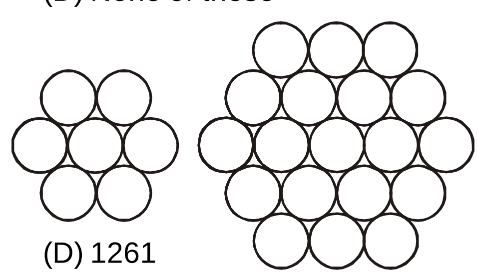
3. The populations (P_A and P_B) of states A and B grow according to $P_A = 3e^{0.05t}$ and $P_B = 5e^{0.03t,}$ where t is the number of years from now and population is in millions. In how many years will state A have twice the population of state B ?

(A) $\frac{\ln(5/6)}{0.02}$ (B) $\frac{\ln(3/10)}{0.02}$ (C) $\frac{\ln(3/5)}{0.02}$ (D) $\frac{\ln(10/3)}{0.02}$

4. How many different pairs of integers (x; y) are solutions of the equation $x^2 - 3y^2 = 1997$?

(A) 1 (B) 2 (C) infinitely many (D) None of these

5. The diagram shows shapes made with the same size coins. The first six-sided shpe has 2 coins along each side, and the second has 3 coins along each side. How many coins do you need to make up a six-sided shape with 21 coins along each side ?



(A) 820 (B) 1071 (C) 1141 (D) 1261

6. The population of a village at one time was perfect square. Later, with an increase of 1`00, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square. The original population is a multiple of

(A) 3 (B) 7 (C) 9 (D) 11

7. In how many different way can 9 oranges be divided among Nic, Sudan and Vishnu in such a way that Nic gets at least 3 oranges, Sudan and Vishnu at least 2 each and Vishnu at most 3 ?
(A) 2 (B) 3 (C) 4 (D) 5

8. Determine the smallest positive value of the integer k such that $k^3 + 2k^2$ is the square of an odd integer.
(A) 14 (B) 23 (C) 7 (D) None of these

9. The number of prime numbers p such that p + 1 is a square is
(A) 1 (B) 4 (C) 3 (D) infinite

10. Let A denotes the set of integers between 1 and 100 which are divisible by 12. Let B denote the set of integers between 1 and 1000 which are divisible by 18. How many elements are in the set $A \cup B$?
(A) 108 (B) 109 (C) 110 (D) 111

11. The largest number of acute angles that a convex hexagon can have, is
(A) 2 (B) 3 (C) 4 (D) 5

12. A farmer has both sheep and chickens. The average number of legs per animal is ℓ. The ratio f the number of sheep to the number of chickens is

(A) $\frac{\ell}{3(4-\ell)}$ (B) $\frac{\ell-2}{4-\ell}$ (C) $\frac{3(\ell-2)}{\ell}$ (D) $\frac{(\ell-2)^2}{16-\ell^2}$

13. It is known that $2^{2^r} + 1$ is prime for r = 0, 1, 2, 3 and 4, but not for r = 5. The number of prime factors of $2^{32} - 1$ is
(A) 1 (B) 6 (C) 3 (D) 5

14. A polygon has n sides all of equal length s. If the area of the polygon is A, then the sum of the shortest distances from any point inside the polygon to each of the sides (produced if necessary) is

(A) $\frac{ns}{2}$ (B) $\frac{A}{ns}$ (C) $\frac{2A}{ns}$ (D) $\frac{2A}{s}$

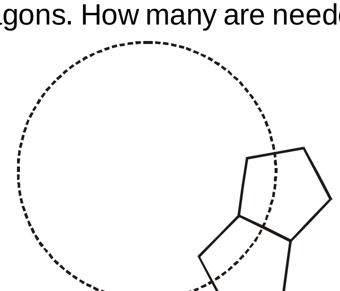
15. Four positive integers a, b, c, d are given. There are exactly 4 distinct ways to choose 3 of a, b, c, d. The mean of each of the four possible triples is added to the 4th integer. The four sums 29, 33, 21, 17 are obtained. One of the original integer is :
(A) 19 (B) 21 (C) 23 (D) 29

16. Let $D = a^2 + b^2 + c^2$ where a and b are consecutive integers and c = ab. Then $\sqrt{D}$ is :
(A) always an even integer (B) sometimes an odd integer, sometimes not
(C) always an odd integer (D) sometimes a rational number, sometimes not

17. The product of the solutions to the quadratic equation $ax^2 + bx + c = 0$ is 6. The product of the solutions of $bx^2 + cx + a = 0$ is 8. What is the product of the solutions of $cx^2 + ax + b = 0$?

(A) $\frac{1}{12}$ (B) $\frac{1}{46}$ (C) $\frac{1}{18}$ (D) $\frac{1}{50}$

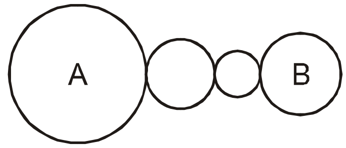
18. Identical regular pentagons are placed together side by side to form a ring in the manner shown. The diagram shows the first two pentagons. How many are needed to make a full ring ?



(A) 9 (B) 10 (C) 11 (D) 12

19. Four wheels with readius 6, 3, 2 and 4 respectively, are pressed together and rotate without slipping.

If wheel A rotates at 60 revolutions per minute, then the speed of wheel B, in revolutions per minute, is



(A) 60 (B) 45 (C) 120 (D) 90

20. In the sequence of fractions $\frac{1}{1},\frac{2}{1},\frac{1}{2},\frac{3}{2},\frac{2}{2},\frac{1}{3},\frac{4}{1},\frac{3}{2},\frac{2}{3},\frac{1}{4},\frac{5}{1},........$, the 2005th fraction is

(A) $\frac{22}{41}$ (B) $\frac{13}{52}$ (C) $\frac{57}{9}$ (D) $\frac{12}{52}$

21. If b is a real number such that $b^2 = b + 1$, then which of the following in NOT true ?
(A) $b^3 = b^2 + b$ (B) $b^4 = b^3 + b + 1$ (C) $b^3 = 2b + 1$ (D) $b^3 + b^2 = b + 1$

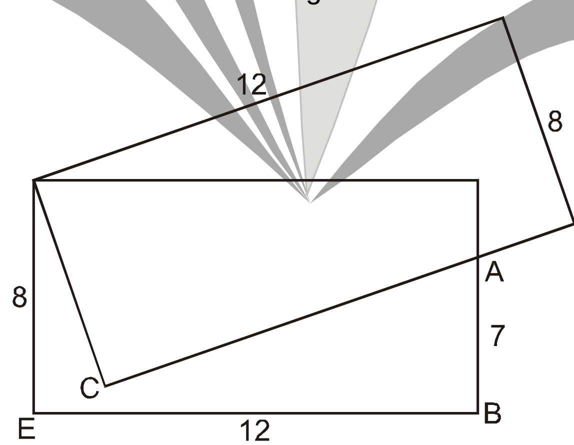
22. Suppose $f(x + 1, y) = f(x, y) + y + 1$, $f(x, 0) = x$ and $f(x, y) = f(y, x)$ for all x and y, then $f(12, 5)$ equals
(A) 77 (B) 60 (C) 17 (D) 83

23. If the difference between two consecutive angles of a convex pentagon is a constant integer, then the number of possible different values of the smallest angle is
(A) 13 (B) 41 (C) 61 (D) 36

24. Find the largest integer value of n so that 3^n divides (25! + 26!).
(A) 10 (B) 11 (C) 12 (D) 13

25. The following is a polynomial : $\sqrt[3]{x^9 - 3x^8 + 18x^7 - 28x^6 + 84x^5 - 42x^4 + 98x^3 + 72x^2 + 15x + 1}$. Find the sum of the squares of the coefficients of this polynomial.
(A) 28 (B) 30 (C) 32 (D) 34

26. Let two 8 × 12 rectangles share a common corner and overlap as in the diagram below, so that the distance AB fromthe bottom right corner of one rectangle to the intersection point A along the right edge of that reactangle is 7. What is the area fo the region common to the two rectangles ?



(A) 40 (B) 42 (C) 44 (D) 46

27. Using the figure from the previous problem. Let the point C be the corner of the slanted rectangle shown. What is the sum of the coordinates of C, given that the lower left corner E of the unslanted rectagle is at (0, 0).
(A) 6.1 (B) 6.2 (C) 6.3 (D) 6.4

28. The sum a + b, the product a . b and the difference of squares $a^2 - b^2$ of two positive numbers a and b is the same nonzero number. What is b?

(A) 1 (B) $\frac{1+\sqrt{5}}{2}$ (C) $\sqrt{3}$ (D) $\frac{7-\sqrt{5}}{2}$

29. A peddler is taking eggs to the market to sell. The eggs are in a cart that holds up to 500 eggs. If the eggs are removed from the cart either 2, 3, 4, 5 or 6 at a time, one egg is always left over. If the eggs are removed 7 at a time, no eggs are left over. Let n denote the number of eggs in the cart. Which of the following is true about n ?

(A) $n \in [1,100]$ (B) $n \in [101, 200]$ (C) $n \in [201, 300]$ (D) $n \in [301, 400]$

30. Let A be the area of a triangle with sides 5, 5 and 8 and let B denote the area of a triangle with sides 5, 5, and 6. Which of the following statements is true ?

(A) A < B < 12 (B) B < A < 12 (C) A = B (D) 12 < A < B

31. Let a, b, c $\geq$ 2 be natural numbers and $a^{(b^c)} = (a^b)^c$? Which one(s) of a, b, c can have arbitrary values.

(A) a (B) b (C) c (D) both a and b

32. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression ?

(A) 1 (B) 4 (C) 36 (D) 49

33. Find the value of the expression S = 1! . 3 – 2! . 4 + 3! . 5 – 4! . 6 + ... – 2006! . 2008 + 2007!

(A) –2007 (B) 0 (C) 1 (D) 2007

34. Suppose N is a positive integer that is a perfect cube. Which of the following represents the next positive integer that is a perfect cube ?

(A) $N^3 + 3\sqrt[3]{N} + 1$ (B) $N + 3\sqrt[3]{N^2} + 3\sqrt[3]{N} + 1$
(C) $N^3 + 3N^2 + 3N + 1$ (D) $N^3 + N^2 + N + 1$

35. Suppose a and b are positive integers for which $(2a + b)^2 – (a + 2b)^2 = 9$. What is ab ?

(A) 2 (B) 6 (C) 9 (D) 12

36. Let x and y be positive integers satisfying $\frac{1}{x+1} + \frac{1}{y-1} = 5/6$. Find x + y.

(A) 2 (B) 3 (C) 4 (D) 5

37. The product of three consecutive positive integers is eight times their sum. What is the sum of their squares ?

(A) 50 (B) 77 (C) 110 (D) 149

38. Which of the following is closest to the smallest positive rational number that is an integer multiple of the numbers $\frac{10}{21}, \frac{5}{14}$ and $\frac{6}{7}$?

(A) 1.43 (B) 2.79 (C) 3.43 (D) 4.29

39. From the list of all natural numbers 2, 3, ...999, delete nine sublists as follows. First, delete all even numbers except 2, then all multiples of 3 except 3, then all multiples of 5 except 5, and so on, for the nine primes 2, 3, 5, 7, 11, 13, 17, 19, 23. Find the sum of the **composite** numbers left is the remaining list.

(A) 0 (B) 899 (C) 961 (D) 3062

40. Let S denote the set {(–2, –2), (2, –2), (–2, 2), (2, 2)}. How many circles of radius 3 in the plane have exactly two points of S on them ?

(A) 6 (B) 8 (C) 10 (D) 12

41. The six-digit number 5ABB7A is a multiple of 33 for digits A and B. Which of the following could be A + B ?

(A) 8 (B) 9 (C) 10 (D) 11

42. What is the smallest positive integer n for which 45n is a perfect cube of an integer ?

(A) 75 (B) 2025 (C) 625 (D) 55

43. Suppose that $2^a + 2^b = 3^c + 3^d$, where all of the exponents are integers. How many of a, b, c, d can be negative ?

(A) 1 (B) 2 (C) 3 (D) None of these

44. Container A, of volume a, is one fifth full. Container B, of volume b, is one sixth full. container C, of volume c, is empty. If all the fluid in the containers is divided equally among the three containers, what part of container C will be full ?

(A) $\frac{6a+5b}{90c}$ (B) $\frac{a+b}{30c}$ (C) $\frac{6a+5b}{30c}$ (D) $\frac{11ab}{90}$

45. The radius of a right circular clinder is increased by 40% and the height is decreased by 50%. What is the change in the volume ?

(A) stay the same (B) increase by 2% (C) decrease by 4% (D) decrease by 2%

46. Let a Ⓛ b represent the operation on two numbers a and b, which selects the larger of the two numbers, with a Ⓛ a = a. Let a Ⓢ b represent the operation which selects the smaller of the two numbers with a Ⓢ a = a. If a, b and c are distincet numbers, and a Ⓢ (b Ⓢ c) = (a Ⓢ b) Ⓛ (a Ⓢ c), then we must have

(A) a < b and a < c (B) a > b and a > c (C) c < b < c (D) c < a < b

47. An ant located at a corner of a 2in. × 3in. × 5in. rectangular block of wood wants to crawl along the surface to the opposite corner of the block. What is the length of the shortest such path ?

(A) $\sqrt{50}$ (B) $\sqrt{58}$ (C) 8 (D) $\sqrt{68}$

48. The product of four ditinct positive integers, a, b, c and d is 8!. The numbers alos satisfy

$$ab + a + b + 1 = 323 \qquad (1)$$
$$bc + b + c + 1 = 399. \qquad (2)$$

What is d ?

(A) 7 (B) 14 (C) 21 (D) 28

49. Consider a sequence $x_1, x_2, x_3, ...$ defined by

$x_1 = \sqrt[3]{3}$, $x_2 = (\sqrt[3]{3})^{\sqrt[3]{3}}$, and in general, $x_n = (x_{n-1})^{\sqrt[3]{3}}$, for n > 1

What is the smallest value of n for which x_n is an integer ?

(A) 3 (B) 4 (C) 9 (D) 27

50. An urn is filled with, coins and beads, all of which are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percent of the object in the urn are gold coins

(A) 40% (B) 48% (C) 52% (D) 60%

ANSWERS

1. B	2. D	3. D	4. D	5. D	6. B	7. D
8. C	9. A	10. D	11. B	12. B	13. D	14. D
15. B	16. C	17. C	18. B	19. D	20. D	21. D
22. A	23. D	24. D	25. A	26. B	27. D	28. B
29. D	30. C	31. A	32. A	33. C	34. B	35. A
36. D	37. B	38. D	39. D	40. D	41. B	42. A
43. D	44. A	45. D	46. A	47. A	48. A	49. B
50. B						